

# Minkowskian Yang-Mills vacuum

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## Abstract

The well-known Bogomol'nyi-Prasad-Sommerfeld (BPS) monopole is considered in the limit of the infinite mass of the Higgs field as a basis for constructing the Yang-Mills vacuum with the finite energy density. In this limit the Higgs field disappears at the spatial infinity, but it leaves, nevertheless, its trace as vacuum Yang-Mills BPS monopoles transformed into Wu-Yang monopoles obtained in the pure Yang-Mills theory by a spontaneous scale symmetry breaking in the class of functions with zero topological charges. The topological degeneration of a vacuum BPS monopole manifests itself via Gribov copies of the covariant Coulomb gauge in the form of the time integral of the Gauss law constraint. We also show that, in the considered theory, there is a zero mode of the Gauss constraint involving an "electric" monopole and the additional mass of the  $\eta'$ -meson in Minkowskian QCD. The consequences of the Minkowskian physical monopole vacuum: rising "golden section" potential and topological confinement, are studied in the framework of the perturbation theory. An estimation of the vacuum expectation value of the square of the magnetic tension is given through the  $\eta'$ -meson mass, and arguments in favour of the stability of the monopole vacuum are considered. We also discuss why all these "smiles" of the Cheshire cat are kept by the Dirac fundamental quantization, but not by the conventional Faddeev-Popov integral.

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# 1 Introduction.

The nature of the vacuum of the Yang-Mills (YM) theory in the Minkowski space still remains an open problem at the present time. There were a lot of attempts to solve this problem.

A typical feature of these attempts was constructing the nontrivial physical vacuum in the Minkowski space on the basis of nonzero values of field vacuum expectations, coinciding with classical fields.

As an example of these attempts we should like to point out the work [1] stimulated by the asymptotic freedom formula, as a criterion for instability of the naive perturbations theory [2].

However, these attempts did not take account of the topological structure of vacuum.

This nontrivial topological structure of the YM vacuum was discovered in the Euclidean space  $E_4$  [3]. This implied that there exist classical *in*- and out-vacuum states corresponding to different topological indices  $|n\rangle$  with zero values of energy and tunnel transitions  $|n\rangle \rightarrow |n+1\rangle$  ( $n \in \mathbf{Z}$ ) occur between them. These transitions are described by *instantons*, i.e. YM fields with fixed topological numbers  $\nu = n_{out} - n_{in}$  on which the YM action attains its minimum corresponding to the zero eigenvalue of energy.

The defects of this vacuum are: the unphysical status of this zero value of energy in quantum theory and explicit violation of the Poincare (CP) invariance.

However, the topological degeneration of initial data for YM fields does not depend on the space where these fields are considered. The initial data of any classical solution in the Minkowski space-time are also topologically degenerated. Therefore it is worth to investigate topologically degenerated vacuum solutions in the Minkowski space-time in the class of functions with physical values of finite energy densities .

In the present paper we attempt to explain the advantages of going over from the Euclidian space-time to the Minkowski one in non-Abelian gauge theories.

This work is devoted to just such investigation of the nontrivial topological vacuum inherent in the YM theory (as a striking pattern of non-Abelian gauge models) in the Minkowski space-time.

The existence of such a vacuum is stipulated by the fact that the homotopies groups of all the three-dimensional paths (loops) in the  $SU(2)$  group manifold is *nontrivial* (see p. 325 in [4]):

$$\pi_3(SU(2)) = \mathbf{Z}. \quad (1.1)$$

We always should take account of this "initial" topology at the analysis of the YM vacuum (independently on the space where we study it).

We investigate the topological degeneration of the initial data using the well-known *Bogomol'nyi-Prasad-Sommerfeld* (BPS) monopole as an example. Such monopoles appear as a result of the spontaneous breakdown of the initial  $SU(2)$  symmetry ( $SU(2) \rightarrow U(1)$ ) in the presence of the Higgs isoscalar multiplet (generalizing the classical Higgs field  $\phi$  in the well-known  $\lambda\phi^4$  theory) in the limit  $\lambda \rightarrow 0$  for the Higgs selfinteraction.

In the lowest order by the temperature  $T$  (effectively, in a neighbourhood of  $T \rightarrow 0$ ) we may always separate in this gas the Bose condensate (at temperatures below

a critical temperature  $T_0 \sim 0$ ) and quantum excitations over this Bose condensate.

Thus there is a possibility to construct the YM vacuum using the Bose condensate of free scalar particles in the limit of their infinite masses,<sup>1</sup> when these particles disappear from the spectrum of elementary excitations of the theory, leaving therein, nevertheless, their various "traces". The study of these "traces" is just the goal of the present paper.

One of these "traces" is the topological degeneration of the BPS monopole perturbation theory, manifesting itself via *Gribov copies* of the covariant Coulomb gauge, treated, in turn, as initial data of the Gauss law constraint in the lowest order of the perturbations theory involving the "new" Minkowskian monopole vacuum. These Gribov copies imply that there exists a zero mode solution to the Gauss law constraint expressed through the *global dynamical variable*  $N(t)$ . This zero mode solution correctly describes the collective solid potential rotation of the (YM-Higgs) Bose condensate with the real energy-momentum spectrum.

We also construct the generating functional for weak perturbation excitations over this vacuum in the form of the Feynman path integral.

The ensuing exposition is organized as follows.

In Section 2 we show that in the Minkowskian YM theory, in the so-called BPS limit  $\lambda \rightarrow 0$ ;  $m \rightarrow 0$  [6] for the Higgs mass and Higgs selfinteraction respectively, there exist nontrivial BPS monopole solutions to the equations of motion involving finite energies densities. They corresponds to the  $SU(2) \rightarrow U(1)$  spontaneous breakdown.

The Bogomol'nyi equation [6], specifying the lowest level of the BPS monopole configuration energy, sets the immediate correlation between YM and Higgs multiplets belonging to the  $U(1) \rightarrow SU(2)$  embedding.

This will be a starting point for the construction of a consistent theory of the Minkowskian YM vacuum in Section 4.

Section 3 is devoted to constructing the Dirac variables in the general YM theory. We demonstrate that they take the look of solutions to the Gauss constraint-shell equation. This will be the base of all our further discussion in the present work.

The topological degeneration of the Minkowskian YM initial data is the subject of Section 4.

We argue, in Subsection 4.1, in favour of that the vacuum in the "old" instanton approach [3] is, indeed, not the physical one.

As an alternative, in Subsections 4.2 and 4.3, we construct "Minkowskian" vacuum monopoles  $\Phi_i^{(0)}(\mathbf{x})$  in the form of a stationary Bose condensate involving topological numbers  $n = 0$  and the nonzero vacuum "magnetic" tension  $B(\Phi_i^{(0)})$  corresponding these YM monopoles.

All this is a result of the  $SU(2) \rightarrow U(1)$  spontaneous breakdown, describing by the classical equations of the Minkowskian non-Abelian theory in the class of fields with

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<sup>1</sup>The difference between the Bose condensate of free scalar particles and Minkowskian YM vacuum is, indeed, in the existence of the (Higgs-YM) interaction with the coupling constant  $g$ . This turns the (Higgs-YM) vacuum into the *c-number Bose condensate in a non-ideal Bose gas, in which inevitable arise long-range correlations of local excitations and cooperative degrees of freedom*. Such a system is alike to the superfluid helium II [5].

topological numbers  $n = 0$ .

These equations permit nontrivial solutions, at the spatial infinity, in the form of Wu-Yang monopoles [7]:  $\Phi_i^{(0)}(\mathbf{x})$ .

Present constructing the Minkowskian YM vacuum is only a presentation of such solutions as BPS monopoles in the theory involving Higgs fields in the BPS limit [6] of their infinite masses (associated with the infinite spatial volume  $V$ ), but at finite energies densities.

Herewith the vacuum "magnetic" tension  $B(\Phi_i^{(0)})$ , specified by the Bogomol'nyi equation [6], itself acquires a crucial importance.

We show that in the considered limit the Gibbs expectation value  $\langle B^2 \rangle$  (specified by means averaging  $B^2$  over the spatial volume) is, indeed, different from zero; in this is an analogy with the *Meisner effect* in a superconductor.

The value  $\langle B^2 \rangle \neq 0$ , inherent in the Minkowskian YM model involving vacuum BPS monopoles, is, in turn, a precondition to solving the  $\eta'$ -meson problem in Minkowskian QCD.

The nonzero value of  $\langle B^2 \rangle$  allows us also to regularize our theory by introducing an infrared cut-off parameter  $\epsilon(\langle B^2 \rangle)$  playing the role of the typical size of BPS monopoles.

The principal goal of Section 4 is to reveal the nature of the topological degeneration of the YM vacuum monopole  $\Phi_i^{(0)}(\mathbf{x})$  and the close correlation between this topological degeneration and the Gribov ambiguity in the choose of the covariant Coulomb gauge (in the form of the time integral of the Gauss law constraint).

This topological degeneration is determined by non-perturbation multipliers  $\exp(n\hat{\Phi}_0(\mathbf{x}))$ , with  $\hat{\Phi}_0(\mathbf{x})$  being a solution to the Gribov ambiguity equation having the look of a Higgs BPS monopole.

In Subsection 4.3 we quote the explicit expression [8] for these gauge transformations, turned YM fields into *topological* Dirac variables satisfied the covariant Coulomb gauge and herewith gauge invariant.

As far as the covariant Coulomb gauge is, indeed, the time integral of the Gauss law constraint, the Gribov ambiguity signals us that there is the zero mode solution to the Gauss law constraint, treated as the equation to the temporal component  $A_0^c(t, \mathbf{x})$  of a YM field.

The in the main new step in our investigations about the Minkowskian YM theory is introducing, in Subsection 4.4, the continuous topological variable  $N(t)$  for specifying the zero mode of the Gauss law constraint. This allows us to represent any Minkowskian YM field  $A_0^c(t, \mathbf{x})$  as the product  $\dot{N}(t)\Phi_0^{(0)c}(\mathbf{x})$ .

This zero mode solution induces the vacuum "electric" tension ("*electric*" *monopole*) as a dynamic degree of freedom that cannot be removed by fixing any gauge.

This "electric" tension, in turn, generates the free rotator action describing the collective solid potential rotation of the Minkowskian (YM-Higgs) vacuum.

The appropriate Schrödinger equation for this vacuum gives the real energy-momentum spectrum, unlike the one in the instanton YM theory [3].

The dependence of the free rotator action on the vacuum expectation value  $\langle B^2 \rangle$ , which, in turn, depends on the Higgs mass (through the Bogomol'nyi equation), confirms

our assumption about the Minkowskian YM vacuum as a Bose condensate.

The topic of Section 5 is a more detailed analysis of the zero mode solution to the Gauss law constraint and the YM (constraint-shell) action; also we decompose the YM "electric" tension into its transverse and longitudinal parts with respect to the constraint-shell (Gauss) equation. We investigate consequences of this decomposing.

Section 6 is devoted to the calculation of the instantaneous potential of the current-current interaction in the presence of Wu-Yang background monopoles [7].

The YM Green function in the Wu-Yang monopoles background takes, indeed, the shape of the sum of two potentials. There are the Coulomb potential and the rising "golden section" one.

This result has a great importance for the analysis of the hadronization and  $\eta'$ -meson problem.

The analysis of the Feynman and FP path integrals is the subject of Section 7.

The last two Sections, 8 and 9, are devoted to the analysis of the infrared topological confinement, involving the quark confinement in QCD as direct consequences of the average over the topological degeneration.

The theory considered in Sections 8 and 9 allow us to assert that only the colourless ("hadronic") states form a complete set of physical states in (Minkowskian) QCD. We prove that the infrared topological confinement implies the quarks confinement in (Minkowskian) QCD, that the complete set of hadronic states ensures that this QCD is, indeed, a unitary theory.

In Section 10 we estimate the value of the vacuum chromomagnetic field in  $\text{QCD}_{(3+1)}$  and point out the way to solve the  $U(1)$ -problem in the Minkowskian YM theory involving vacuum BPS monopoles.

## 2 Gauge Higgs effect in Minkowski space.

Our idea, basing onto our discussion in the previous subsection, is to construct the physical Minkowskian YM vacuum using the Higgs Bose condensate in the Minkowskian YM theories involving monopoles [6, 9, 10].

Herewith we desire to utilize the well-known *Bogomol'nyi-Prasad-Sommerfeld* (BPS) *limit of the zero self-interaction*:  $\lambda \rightarrow 0$  (at  $m \rightarrow 0$ ) in the Higgs sector of the Minkowskian YM action (see, e.g., [4, 11])

$$S = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^b F_b^{\mu\nu} + \frac{1}{2} \int d^4x (D_\mu \phi, D^\mu \phi) - \frac{\lambda}{4} \int d^4x \left[ (\phi^b)^2 - \frac{m^2}{\lambda} \right]^2, \quad (2.1)$$

with

$$D_\mu \phi = \partial^\mu \phi + g[A^\mu, \phi]$$

being the YM covariant derivative and  $g$  being the YM coupling constant.

We suppose that the initial data of all the fields are given to within stationary gauge transformations, the manifold of these transformations has a nontrivial structure of three-

dimensional paths in the group space of the (initial) non-Abelian  $SU(2)$  gauge group:

$$\pi_3(SU(2)) = \mathbf{Z}, \quad (2.2)$$

with  $\mathbf{Z}$  being the group of integers:  $n = 0, \pm 1, \pm 2, \dots$

In the case of the  $SU(2)$  gauge theory the Yang-Mills fields  $A^{\mu b}$  and Higgs fields  $\phi^b$  take their values in the Lie algebra of the  $SU(2)$  group.

To obtain the converging action integral, corresponding to finite values of energy, we should claim for the Higgs field  $\phi(\mathbf{r})$  to be finite as  $\mathbf{r} \rightarrow \infty$  in the BPS limit  $\lambda \rightarrow 0$ .

This implies that  $\phi^a$  would go to the minimum of the potential  $V \equiv \frac{\lambda}{4}(\frac{m^2}{\lambda} - \phi^2)^2$ :

$$\phi^{a\infty}(\mathbf{n}) \in M_0, \quad \mathbf{n} = \frac{\mathbf{r}}{r}, \quad (2.3)$$

where  $M_0$  is the manifold of the minimum of the potential  $V$  (*the vacuum manifold*):

$$M_0 = \{\phi = a; \quad a^2 = m^2/\lambda\} \quad (2.4)$$

as  $\mathbf{r} \rightarrow \infty$ . Thus  $M_0$  consists of the points of the two-sphere  $S^2$  in the three-dimensional  $SU(2)$  group space.

The presence of Higgs *pseudo-Goldstone* modes [12] implies that the initial  $SU(2)$  gauge symmetry inherent in the (Minkowskian) YM model is then spontaneously violated down to its  $U(1)$  subgroup (via the Higgs mechanism of the  $SU(2)$  gauge symmetry breakdown: see, e.g., pp. 243- 244 in [13]).

On the face of it, we may choose the Higgs isovector  $\vec{\phi}$  along the axis  $z$  in the Cartesian coordinates:

$$\vec{\phi} = (0, 0, m/\sqrt{\lambda}), \quad (2.5)$$

as a ground state configuration. Thus this vector stays invariant under rotations around the axis  $z$  ( $U(1)$  transformations).

Note, however, that the choice (2.5) in the whole space is, indeed, *topologically trivial*. Really (see [4], §Φ4), the gauge condition

$$\phi_i = 0, \quad i = 1, 2; \quad \phi_3 = |\vec{\phi}|$$

is not compatible with nontrivial topologies  $n \neq 0$ . For the field satisfying such gauge condition its asymptotic at the spatial infinity is trivial: the solutions of the look

$$\vec{\phi}^\infty(\mathbf{n}) = \mathbf{V}(\mathbf{n})\vec{\phi}, \quad (2.6)$$

with  $V(\mathbf{n})$  being a continuous function of  $\mathbf{n}$  ( $\mathbf{n} = \mathbf{r}/r$  is the unit radius of the spatial sphere  $S^2$ ) taking its values in the  $SU(2)$  group space in the case of the YM theory, are topologically equivalent to  $\vec{\phi} = (0, 0, a)$ . On the other hand,  $\mathbf{V}(\mathbf{n})$ , considered as the map  $\pi_2(SU(2))$ , is equal to zero.

We should define the topological structure of the manifold (2.4).

In the case of a discrete group  $G$ ,  $\phi^{a\infty}$  would be constant, as long as it is a continuous function (from the topological point of view, we deal in this case with the group  $\pi_0$  [15])

of connection components, that is trivial in the case of a connected manifold; the sphere  $S^2 := \{\mathbf{n} = 1 \text{ as } \mathbf{r} \rightarrow \infty\}$  is an example of such manifolds). In this case  $\phi^{a\infty}$  has a trivial topology.

If  $\dim(M_0) \neq 0$ ,  $M_0$  has a nontrivial topology. Therefore the group of symmetry  $G$  *would be continuous*. One may be shown (see, e.g. pp. 465- 466 in [13]) that the covariant derivatives  $D_i\phi$ , entering the action (2.1), decreases as  $r^{-2}$ ; thus the integral (2.1) is, indeed, finite. This guarantees nontrivial topological features of the Minkowskian YM theory.

Issuing from Eq. (2.4), that specifies the manifold  $M_0$  of the minimum of the potential  $V$ , we see that the sphere  $S^2 \simeq M_0$  maps into the sphere  $S^2 := \{\mathbf{n} = 1\}$  as  $\mathbf{r} \rightarrow \infty$ . This map has the nontrivial homotopies group of two-dimensional loops:

$$\pi_2 S^2 = \pi_3(SU(2)) = \pi_1(U(1)) = \pi_1 S^1 = \mathbf{Z}. \quad (2.7)$$

Just this nontrivial topology determines *magnetic charges* associated with the residual  $U(1)$  gauge symmetry (these charges alone point to an "electromagnetic" theory). The presence of magnetic charges implies that there exists the solution to the equations of motion for the YM action (2.1) *in the class of magnetic monopoles*, i.e. *the stationary vacuum solutions at the spatial infinity corresponding to the quantum-field configuration of the minimum energy,  $E_{min}$*  (according to our definition of the vacuum as a ground state of the minimum energy). We may write down this monopole solution.

In particular, the Higgs isovector would be directly proportional to  $\mathbf{n}$  as  $\mathbf{r} \rightarrow \infty$ : in the light of the said above about the map  $S^2 \simeq M_0 \rightarrow S^2 := \{\mathbf{n} = 1\}$  as  $\mathbf{r} \rightarrow \infty$ . Thus its look would be

$$\phi^a \sim \frac{x^a}{r} f(r, a) \quad (2.8)$$

as  $\mathbf{r} \rightarrow \infty$ ;  $f(r, a)$  is a continuous function that does not change the topology (2.7).

This solution for  $\phi^a$  appears for the first time in the work [10], and it is called the *hedgehog*. A good analysis of hedgehogs also is carried out in the monograph [14] (pp. 114- 116).

There may be shown (see §Φ11 in [4]) that there exists the (vacuum) solution to the equations of motion (in the zero topological sector of the Minkowskian YM theory), *regular in a finite spatial volume* and generalizing Polyakov hedgehogs (2.8) <sup>2</sup>, in the form [4, 11]

$$\phi^a = \frac{x^a}{gr} f_0^{BPS}(r), \quad f_0^{BPS}(r) = \left[ \frac{1}{\epsilon \tanh(r/\epsilon)} - \frac{1}{r} \right], \quad (2.9)$$

$$A_i^a(t, \vec{x}) \equiv \Phi_i^{aBPS}(\vec{x}) = \epsilon_{iak} \frac{x^k}{gr^2} f_1^{BPS}(r), \quad f_1^{BPS} = \left[ 1 - \frac{r}{\epsilon \sinh(r/\epsilon)} \right], \quad (2.10)$$

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<sup>2</sup>The statement that the said solution is regular in a finite spatial volume implies that we should consider the topology (2.7) and the vacuum manifold  $M_0$ , (2.4), taking account of this finite spatial volume. If we wish to adapt our theory to the needs of (Minkowskian) QCD (we shall see how to do this in Sections 8 and 9), the spatial volume specified by the typical hadronic size,  $\sim 1$  fm. ( $\sim 5 \text{ GeV}^{-1}$ ), is quite sufficient for our purposes.

obtained in the BPS limit

$$\lambda \rightarrow 0, \quad m \rightarrow 0 : \quad \frac{1}{\epsilon} \equiv \frac{gm}{\sqrt{\lambda}} \neq 0. \quad (2.11)$$

The Higgs Bose condensate behaves as the Bose condensate in an ideal gas in this limit.

The functions  $f_0^{BPS}(r)$  and  $f_1^{BPS}(r)$  are called the *BPS ansatzes*, while the solutions (2.9), (2.10) are called **BPS monopoles**.

The vacuum solution (2.9), (2.10) satisfies the *potentiality condition*:

$$\mathbf{B} = \pm D\vec{\phi}, \quad (2.12)$$

with  $\mathbf{B}$  being the vacuum "magnetic" tension in the theory (2.1). This equation (called *the Bogomol'nyi equation*) is obtained at evaluating the lowest bound of the (YM-Higgs) energy:

$$E_{min} = 4\pi \mathbf{m} \frac{a}{g}, \quad a = \frac{m}{\sqrt{\lambda}} \quad (2.13)$$

(with  $\mathbf{m}$  being the magnetic charge), for the BPS monopole solutions.

The Bogomol'nyi equation (2.12) shows that there exists a nonzero vacuum "magnetic" tension  $\mathbf{B}$  in the Minkowskian YM theory involving vacuum BPS monopoles in its Higgs and YM sectors.

The Bogomol'nyi equation (2.12) may be rewritten in the tensor form as [4]

$$\frac{1}{2g} \epsilon^{ijk} F_{jk} = \nabla^i \phi. \quad (2.14)$$

We see that the Bogomol'nyi equation (2.12) allows us to get a consistent theory involving YM and Higgs  $U(1) \rightarrow SU(2)$  multiplets and yielding the solutions of the BPS monopole type. The Higgs sector of that theory defines the  $U(1)$  group of gauge symmetry with the nontrivial topology (2.7), involving magnetic charges and radial "magnetic" (vacuum) fields.

The said may serve as a good base for constructing the consistent theory for the physical YM vacuum in the Minkowski space.

In contrast to the "old" Euclidian approach [3] to the YM vacuum, our "Minkowskian" conception of the YM vacuum as a stationary Bose condensate with the simultaneous strong (YM-Higgs) coupling [16] yields, indeed, the real spectrum of momentum.

We shall make sure that the stationary vacuum fields in the new Minkowskian YM theory possess the winding numbers  $n = 0$  and they indeed have the look of BPS monopoles, (2.9), (2.10) respectively.

The "electric" and "magnetic" tensions corresponding to these vacuum fields also will be constructed.

The topological degeneration (for  $n \neq 0$ ) in the theory here represented is realized via *Gribov copies* of the covariant Coulomb gauge imposed on (vacuum) YM potentials.

The YM (gluonic) fields are considered as weak perturbation excitations (*multipoles*) over this BPS monopole vacuum. These excitations have the asymptotic  $O(\frac{1}{r^{l+1}})$ ,  $l > 1$  at the spatial infinity.

All this will be discussed in the next sections.



### 3 Dirac quantization of Yang-Mills theory.

Let us consider the "pure" YM theory involving the gauge  $SU(2)$  group in the four-dimensional Minkowski space-time. The action of that theory is given by the formula

$$W[A_\mu] = -\frac{1}{4} \int d^4x F_{\mu\nu}^a F_a^{\mu\nu} = \frac{1}{2} \int d^4x (F_{0i}^{a2} - B_i^{a2}), \quad (3.1)$$

where the standard definitions of the non-Abelian "electric" tension  $F_{0i}^a$ :

$$F_{0i}^a = \partial_0 A_i^a - D(A)_i^{ab} A_{0b}, \quad D_i^{ab} = (\delta^{ab} \partial_i + g \epsilon^{acb} A_{ci}), \quad (3.2)$$

and the "magnetic" one,  $B_i^a$ :

$$B_i^a = \epsilon_{ijk} (\partial^j A^{ak} + \frac{g}{2} \epsilon^{abc} A_b^j A_c^k), \quad (3.3)$$

are used. The action (3.1) is invariant with respect to the gauge transformations  $u(t; \mathbf{x})$ :

$$\hat{A}_i^u = u(t; \mathbf{x}) (\hat{A}_i + \partial_i) u^{-1}(t, \mathbf{x}), \quad (3.4)$$

with  $\hat{A}_\mu = g \tau_{2i}^a A_{a\mu}$ .

Solutions to the non-Abelian constraint equation (*the Gauss law constraint*):

$$\frac{\delta W}{\delta A_0^a} = 0, \iff [D^2(A)]^{ac} A_{0c} = D_i^{ac}(A) \partial_0 A_c^i, \quad (3.5)$$

and to the equation of motion:

$$\frac{\delta W}{\delta A_i^a} = 0, \iff [\delta_{ij} D_k^2(A) - D_j(A) D_i(A)]^{ac} A_c^j = D_0^{ac}(A) [\partial^0 A_{ci} - D(A)_{cbi} A^{ob}], \quad (3.6)$$

are specified by boundary conditions and initial data. They generalize the appropriate equations in the Maxwell electrodynamics (see Eqs. (7), (8) in [17]).

The Gauss law constraint (3.5) associates initial data of  $A_a^0$  to the one of the spatial components  $A_a^i$ .

To remove the unphysical variables, we may solve this constraint in the form of the naive perturbation series:

$$A_c^0 = \frac{1}{\Delta} \partial_0 \partial_i A_c^i + \dots, \quad (3.7)$$

with  $\Delta$  being the Laplacian. As we remember from mathematical physics (see, e.g., p. 203 in [18]), the *fundamental solution* to the *Laplace equation*:

$$\Delta \mathcal{E}_3 = \delta(x), \quad (3.8)$$

is

$$\mathcal{E}_3 = -\frac{1}{4\pi x}. \quad (3.9)$$

This specifies the action of the operator  $\Delta^{-1}$  on a continuous function  $f(x)$ :

$$\Delta^{-1}f(x) = -\frac{1}{4\pi} \int d^3y \frac{f(y)}{|x-y|}, \quad (3.10)$$

with  $\Delta^{-1}$  being the *Coulomb kernel of the non-local distribution* (see also (12) in [17]).

Thus resolving the (YM) Gauss law constraint and substituting this solution into the equations of motion distinguishes gauge invariant non-local (radiation) variables.

Upon substituting this solution into Eq. (3.6) the lowest order of this equation in the coupling constant  $g$  contains *only transverse fields* (this level mathematically coincides, as a *linearized YM theory*, with the theory of radiation variables in QED [17]):

$$[\partial_0^2 - \Delta]A_k^{cT} + \dots = 0, \quad A_i^{cT} = [\delta_{ik} - \partial_i \Delta^{-1} \partial_k]A^{ck} + \dots \quad (3.11)$$

This perturbation theory is well known as the *radiation* [19] or *Coulomb* [20, 21] gauge involving the generating functional of Green functions in the form of a Feynman integral in the rest reference frame  $l^{(0)} = (1, 0, 0, 0)$ :

$$\begin{aligned} Z_F[l^{(0)}, J^{aT}] &= \int \int \prod_{c=1}^{c=3} [d^2 A^{cT} d^2 E_{cT}] \\ &\times \exp \left\{ i W_{l^{(0)}}^T[A^T, E^T] - i \int d^4x [J_k^{cT} \cdot A_{cT}^k] \right\}, \end{aligned} \quad (3.12)$$

with the constraint-shell action:

$$W_{l^{(0)}}^T[A^T, E^T] = W^I|_{\frac{\delta W^I}{\delta A_0}=0}, \quad (3.13)$$

given in the first order formalism (see [22], p. 83):

$$W^I = \int dt \int d^3x \{ F_{0i}^c E_c^i - \frac{1}{2} [E_i^c E_c^i + B_i^c B_c^i] \} = \int dt \int d^3x (E^{ci} \partial_0 A_{ci} + A_{0c} D^c - H), \quad (3.14)$$

where

$$D^c = \partial_k E^{kc} - g[A_k^b, E^{kd}] \epsilon_{bd}^c, \quad (3.15)$$

and

$$H = \frac{1}{2} (E_k^{c2} + B_k^{c2}) = \frac{1}{2} [(E^{Tc})^2 + (\partial_i \sigma^c)^2 + B_k^{c2}] \quad (3.16)$$

is the Hamiltonian of the YM theory.

We decompose here the "electric" tension  $E^{kc}$  into the *transverse and longitudinal parts* respectively:

$$E_i^c = E_i^{Tc} + \partial_i \sigma^c, \quad \partial_i E_i^{Tc} = 0. \quad (3.17)$$

The constraint

$$\frac{\delta W^I}{\delta A_0} = 0 \iff D_i^{cd}(A) E_d^i = 0 \quad (3.18)$$

may be solved in terms of *radiation* variables.

The function  $\sigma^a$  has the look [20]

$$\sigma^a[A^T, E^T] = \left(\frac{1}{D_i(A)\partial^i}\right)^{ac} \epsilon_{cbd} A_k^{Tb} E^{Tkd} \equiv \left(\frac{1}{\hat{\Delta}}\right)^{ac} \epsilon_{cbd} A_k^{Tb} E^{Tkd}. \quad (3.19)$$

It is worth to notice that the vector  $D^c$  in (3.15), that is the nothing else as the contrvariant derivative of the "electric" field  $E_i^c$ , disappears, indeed, on the surface of the Gauss law constraint (3.18).

Note also (see Eq. (16.24) in [21]) that  $\det[D_i(A)\partial^i]$  in (3.19) is precisely *the Faddeev-Popov (FP) determinant* in the YM Hamiltonian formalism corresponding to the transverse (Coulomb) gauge of YM fields:

$$\hat{\Delta}_a^b A_{0b} - \partial_i E_a^i = 0, \quad \hat{\Delta}_a^b \equiv D_{ai}^b \partial^i, \quad (3.20)$$

with

$$E_{ia} = \frac{\partial \mathcal{L}}{\partial \dot{A}^{ia}} = F_{0i}^a \quad (3.21)$$

being the canonical momentum (3.2).

A complete proof that  $\det \hat{\Delta}_a^b$  is, indeed, the FP determinant of the YM theory is given in the monograph [21], where there was shown that the radiation gauge in the YM theory is equivalent to the FP determinant  $\det \hat{\Delta}_a^b$  (see (16.30) in [21]).

The operator quantization of the YM theory in terms of the radiation variables belongs to Schwinger [19], who proved the relativistic covariance of the radiation variables (3.11).

This implies that the radiation fields are transformed as non-local functionals (*Dirac variables* [17]),

$$\hat{A}_k^T[A] = v^T[A](\hat{A}_k + \partial_k)(v^T[A])^{-1}, \quad \hat{A}_k^T = g \frac{A_k^{Ta} \tau_a}{2i}, \quad (3.22)$$

where matrices  $v^T[A]$  are specified issuing from the claim to YM fields to be transverse:  $\partial_k A^{kT} = 0$ .

These matrices also would cancel the action of the  $SU(2)$  gauge group, (3.4), onto YM fields, to ensure the gauge invariance of Dirac variables. More precisely, matrices  $v^T[A]$  would be transformed as

$$v^T[A] \rightarrow v_u^T[A] = u^{-1}(t, \mathbf{x}) v^T[A] \quad (3.23)$$

at the gauge transformations (3.4) [23].

It is less obviously that the Dirac variables (3.22) are actually transverse. A good analysis of this fact was carried out in the review [17] it is the linearized YM theory (3.7), (3.11).

One may be shown, utilizing the arguments of the Dirac variables analysis in QED [17] (see also [8, 16, 23]), that the Dirac variables (3.22) are transverse if and only if the Dirac matrices  $v^T[A]$  have the look

$$v^T(t, \mathbf{x}) = v^T(\mathbf{x}) T \exp\left\{\int_{t_0}^t d\bar{t} \hat{A}_0(\bar{t}, \mathbf{x})\right\}; \quad v^T(t, \mathbf{x})|_{t=t_0} = v^T(\mathbf{x}) \quad (3.24)$$

(the symbol  $T$  denotes the time ordering of the matrices under the exponent sign), where, resolving the Gauss law constraint (3.5), we represent the temporal component of an YM field,  $\hat{A}_0$ , as the series (3.7) in the linearized YM theory, i.e. as a non-local functional of YM fields. Just this proves that the Dirac variables (3.22) are indeed transverse.

Our discussion about the Dirac matrices  $v^T[A]$  we shall continue in Section 4.3. We shall show that the topological degeneration of initial (Minkowskian) YM data is determined only by stationary matrices  $v^T(\mathbf{x})$ . The latter one comes to Gribov topological factors [24]  $v^{(n)}(\mathbf{x}) \equiv \exp(n\hat{\Phi}_0(\mathbf{x}))$ , with  $n \in \mathbf{Z}$  and  $\hat{\Phi}_0(\mathbf{x})$  being the so-called Gribov phase, whose explicit look we shall ascertain in Section 4.3 [8, 17, 25]. Ibid we also shall generalize the linearized YM theory (3.7) and rewrite the exponential multiplier in (4.3) in the form of the so-called "Dirac dressing" [8, 17] of non-Abelian fields, depending explicitly on Minkowskian vacuum YM solutions: BPS and Wu-Yang monopoles.

Note that the removal [26] of temporal components of YM fields (contradicting the Dirac quantization principles) implies that the appropriate "temporal" Dirac variables are equal to zero [23]:

$$v^T[A](A_0(t) + \partial_0)(v^T)^{-1}[A] = 0. \quad (3.25)$$

We may treat the latter formula as an equation for specifying Dirac matrices  $v^T[A]$ .

At the level of the Feynman path integral the relativistic covariance implies a relativistic transformation of sources (this, in turn, involves the appropriate relativistic transformations of the Dirac variables (3.22) [17]).

The definition (3.22) of the Dirac variables may be interpreted as a transition to new variables, allowing us to rewrite the Feynman integral (3.12) in the form of the appropriate FP integral [20, 27, 28]:

$$\begin{aligned} Z_F[l^{(0)}, J^{aT}] &= \int \int \prod_{c=1}^{c=3} [d^4 A^c] \delta(\partial_i A^{ci}) \text{Det}[D_i(A) \partial^i] \\ &\times \exp \left\{ iW[A] - i \int d^4 x (J_{ck}^T \cdot A^{Tkc}[A]) \right\}. \end{aligned} \quad (3.26)$$

It was proved in [20, 27, 28] that, *on mass-shells of radiation fields, the scattering amplitudes do not depend on factors  $v^T[A]$* . But the following question is quite reasonable: *why we cannot observe these scattering amplitudes?* There are a few answers to this question: the infrared instability of the naive perturbation theory [1, 29], the Gribov ambiguity, or the zero value of the FP determinant [24], the topological degeneration of the physical states [30, 31, 32].

This will be the subject of our discussion in the next sections.

## 4 Topological degeneration of initial data.

### 4.1 Instanton theory.

One may find a lot of solutions to equations of classical Maxwell electrodynamics. Nature chooses, however, two types of functions: *monopoles* (electric charges), that specify non-local electrostatic phenomena (including instantaneous bound states) and *multipoles*, that specify spatial components of gauge fields involving nonzero magnetic tensions.

Spatial components of non-Abelian fields, considered above as the radiation variables (3.11) in the naive perturbation theory (3.7), are also defined as multipoles.

In the non-Abelian theory, however, it is a reason, as we have seen this in Section 2, to assume that *spatial components of non-Abelian fields*: for example, YM vacuum BPS monopoles (2.10), *belong to the monopole class of functions*, like temporal components of Abelian fields (as, for instance, the Coulomb potential), that we also treat as monopoles.

This fact was revealed by the authors of the instanton YM theory [3]. Instantons satisfy the duality equation in the Euclidean space  $E_4$  (where the Hodge duality operator  $*$  has the  $\pm 1$  eigenvalues for external 2-forms defining the YM tension tensor); thus the instanton YM action coincides, in effect, with the *Chern-Simons functional* (the *Pontryagin index*) (see, e.g., Eq. (10.104) in [33]):

$$\nu[A] = \frac{g^2}{16\pi^2} \int_{t_{in}}^{t_{out}} dt \int d^3x F_{\mu\nu}^a * F_a^{\mu\nu} = X[A_{out}] - X[A_{in}] = n(t_{out}) - n(t_{in}), \quad (4.1)$$

with (see (10.93) in [33])

$$X[A] = -\frac{1}{8\pi^2} \int_V d^3x \epsilon^{ijk} \text{tr}[\hat{A}_i \partial_j \hat{A}_k - \frac{2}{3} \hat{A}_i \hat{A}_j \hat{A}_k]; \quad A_{in,out} = A(t_{in,out}, x); \quad (4.2)$$

being the *topological winding number functional of gauge fields* and  $n \in \mathbf{Z}$  being the value of this functional for the classical vacuum:

$$\hat{A}_i = L_i^n = v^{(n)}(\mathbf{x}) \partial_i v^{(n)}(\mathbf{x})^{-1}. \quad (4.3)$$

The manifold of all the classical vacua in a non-Abelian theory represents *the group of three-dimensional paths* lying in the three-dimensional  $SU(2)$ -manifold with the homotopies group  $\pi_3(SU(2)) = \mathbf{Z}$ .

The whole group of stationary matrices  $v^{(n)}(\mathbf{x})$  is split into the topological classes marked by the integer topological numbers (the *Pontryagin degrees of the map*) specified by the expression (see (10.106) in [33])

$$\mathcal{N}[n] = -\frac{1}{24\pi^2} \int d^3x \epsilon^{ijk} \text{tr}[L_i^n L_j^n L_k^n], \quad (4.4)$$

that shows [8] how many times a three-dimensional path  $v^{(n)}(\mathbf{x})$  turns around the  $SU(2)$  group manifold when the coordinate  $x_i$  runs over the space where it is specified.

Gribov, in 1976, proposed to consider instantons as *Euclidean solutions interpolating between classical vacua with different degrees of the map* (or as *tunnel transitions between these classical vacua*).

The degree of the map (4.4) may be treated as a normalization condition that specifies the class of functions for the given classical vacuum (4.3).

In particular, to obtain Eq. (4.3), we should choose the classical vacuum in the form

$$v^{(n)}(\mathbf{x}) = \exp(n\hat{\Phi}_0(\mathbf{x})), \quad \hat{\Phi}_0 = -i\pi \frac{\tau^a x_a}{r} f_0(r) \quad (r = |\mathbf{r}|) \quad (4.5)$$

(compare with (16.34) in [13]; we should also set  $x_0 = 0$  in this formula for stationary gauge transformations).

The function  $f_0(r)$  satisfies the boundary conditions

$$f_0(0) = 0, \quad f_0(\infty) = 1. \quad (4.6)$$

Note a parallel between this solution and Eq. (2.8).

The common between the Minkowskian monopole and Euclidian instanton YM theories is, indeed, *in the similar topological structure inherent in the both theories*.

In the case of the YM instanton theory [3] we deal with the map (2.2):  $S^3 \rightarrow SU(2)$  as  $\mathbf{x} \rightarrow \infty$ . This induces the homotopies group  $\pi_3(SU(2)) = \pi_3 S^3 = \mathbf{Z}$  (where  $S^3$  is the bound of the Euclidian space  $E_4$ ) coinciding with  $\pi_2 S^2 = \mathbf{Z}$  (see (2.7)) in the Minkowskian theory (2.1)- (2.12), involving vacuum BPS monopoles in the YM and Higgs sectors of that theory.

This generates similar theories.

But there exists also the principal distinction between the both theories.

As a consequence of the relation

$$\pi_3(SU(2)) = \pi_3 S^3 = \mathbf{Z},$$

instantons **may exist in the Euclidian YM theory [3] without any spontaneous  $SU(2)$  breakdown**. This breakdown is not a necessary thing in this case, and we consider  $SU(2)$  as an *exact symmetry* in the instanton YM theory [3].

Thus we obtain the solution of the monopole type in (4.3) as  $\mathbf{x} \rightarrow \infty$ .

To show that these classical values are not sufficient to describe a physical vacuum in the non-Abelian (Euclidian) theory, we should consider a *quantum instanton*, i.e. a corresponding to the zero vacuum solution to the Schrödinger equation [8, 16]

$$\hat{H}\Psi_0[A] = 0, \quad (4.7)$$

with  $\hat{H} = \int d^3x [E^2 + B^2]$ ,  $E = \frac{\delta}{i\delta A}$  being the operators of the Hamiltonian and field momentum respectively. This solution may be constructed by using the winding number functional (4.2) and its variation derivative,

$$\frac{\delta}{\delta A_i^c} X[A] = \frac{g^2}{16\pi^2} B_i^c(A). \quad (4.8)$$

The vacuum wave functional, rewriting in terms of the winding number functional (4.2), has the look of a plane wave [16]:

$$\Psi_0[A] = \exp(iP_{\mathcal{N}}X[A]), \quad (4.9)$$

for *unphysical* imaginary values of the topological momentum  $P_{\mathcal{N}} = \pm 8\pi i/g^2$  [16, 34].

We should like to note that in QED this type of the wave functional belongs to the unphysical part of the spectrum, like the wave function of the harmonious oscillator  $(\hat{p}^2 + q^2)\phi_0 = 0$ . The value of this unphysical plane wave functional <sup>3</sup> for the classical vacuum (4.3) coincides with the *semi-classical instanton wave function*

$$\exp(iW[A_{\text{instanton}}]) = \Psi_0[A = L_{\text{out}}] \times \Psi_0^*[A = L_{\text{in}}] = \exp(-\frac{8\pi^2}{g^2}[n_{\text{out}} - n_{\text{in}}]). \quad (4.10)$$

This exact relation between a semi-classical instanton and its quantum version (4.7) points out that classical YM instantons [3] are also unphysical solutions; they permanently tunnel in the Euclidean space-time  $E_4$  between the classical vacua with zero energies that do not belong to the physical spectrum.

## 4.2 Physical vacuum and gauge Higgs effect.

Our next step is the assertion [35] about the topological degeneration of initial data not only of the classical vacuum but also of all the physical fields with respect to the stationary gauge transformations

$$\hat{A}_i^{(n)}(t_0, \mathbf{x}) = v^{(n)}(\mathbf{x})\hat{A}_i^{(0)}(t_0, \mathbf{x})v^{(n)}(\mathbf{x})^{-1} + L_i^n, \quad L_i^n = v^{(n)}(\mathbf{x})\partial_i v^{(n)}(\mathbf{x})^{-1}. \quad (4.11)$$

The stationary transformations  $v^{(n)}(\mathbf{x})$  with  $n = 0$  are called *the small one*; and those with  $n \neq 0$  are called *the large ones* [35].

The group of transformations (4.11) implies that *spatial components of non-Abelian fields, involving nonzero magnetic tensions  $B(A) \neq 0$  belong to the monopole class of functions*, like temporal components of Abelian fields. In this case non-Abelian fields involving nonzero magnetic tensions contain non-perturbation monopole-type terms, and spatial components may be decomposed [16] into sums of vacuum monopoles  $\Phi_i^{(0)}(\mathbf{x})$  and multipoles  $\bar{A}_i$ :

$$A_i^{(0)}(t_0, \mathbf{x}) = \Phi_i^{(0)}(\mathbf{x}) + \bar{A}_i^{(0)}(t_0, \mathbf{x}). \quad (4.12)$$

Each multipole is treated as a weak perturbation over the appropriate monopole having the following asymptotic at the spatial infinity [8]:

$$\bar{A}_i(t_0, \mathbf{x})|_{\text{asymptotic}} = O(\frac{1}{r^{l+1}}) \quad (l > 1). \quad (4.13)$$

---

<sup>3</sup>The wave function (4.9) is not good normalized, the imaginary topological momentum  $P_{\mathcal{N}} = \pm 8\pi i/g^2$  turns it in a function with the non-integrable square.

Nielsen and Olesen [29] and Matinyan and Savidy [1] introduced the vacuum "magnetic" tension using the fact that all the asymptotically free theories are instable, and the perturbation vacuum is not the lowest stable state.

The extension of the topological classification of classical vacua to all the initial data of the spatial components helps us to choose vacuum monopoles with the zero value of the winding number functional (4.1):

$$X[A = \Phi_i^{c(0)}] = 0, \quad \frac{\delta X[A]}{\delta A_i^c} \Big|_{A=\Phi^{(0)}} \neq 0. \quad (4.14)$$

The zero value of the winding number functional, the claim for non-Abelian fields to be transverse, and the spherical symmetry of vacuum monopoles fix the class of initial data for spatial components:

$$\hat{\Phi}_i = -i \frac{\tau^a}{2} \epsilon_{iak} \frac{x^k}{r^2} f(r). \quad (4.15)$$

They contain an unknown function  $f(r)$ . The classical equation for this function takes the form [8]

$$D_k^{ab}(\Phi_i) F_a^{bk}(\Phi_i) = 0 \implies \frac{d^2 f}{dr^2} + \frac{f(f^2 - 1)}{r^2} = 0. \quad (4.16)$$

One may see three solutions to this equation:

$$f_1^{PT} = 0, \quad f_1^{WY} = \pm 1 \quad (r \neq 0). \quad (4.17)$$

The first solution corresponds to the naive instable perturbation theory, involving the asymptotic freedom formula [1, 2].

Two nontrivial solutions  $f_1^{WY} = \pm 1$  are also well known. They are the Wu-Yang monopoles [7], applied to construct physical variables in the work [36] (the hedgehog and the antihedgehog, respectively, in the terminology [14, 10]).

As it was shown in the papers [8, 17, 23, 25] (and we shall repeat these arguments in Section 6), Wu-Yang monopoles involve the rising potential of the instantaneous interaction between two total [25] excitation currents. This interaction rearranges the perturbation series, leads to the appearance of the gluonic constituent mass and removes the asymptotic freedom formula [37, 38] as an origin of instability in the non-Abelian model considered.

The appearance of said two solutions with the opposite signs in (4.17) is very remarkable, especially as we approximate our BPS monopole solutions by Wu-Yang monopole ones; it is easy to see that we obtain then the BPS ansatzes  $\pm f_1^{BPS}(r)$  in (2.10) instead of  $f_1^{WY}$  in (4.17).

Wu-Yang monopoles [7], thus, are solutions to the classical equations everywhere *excepting an infinite small neighbourhood of the origin of coordinates*,  $r = 0$ . The appropriate "magnetic" field is

$$B_i^a(\Phi_k) = \frac{x^a x^i}{gr^4}. \quad (4.18)$$



To remove this singularity of Wu-Yang monopoles [7] at the origin of coordinates and to regularize its energy, Wu-Yang monopoles [7] are considered as the limit (as  $r \rightarrow \infty$ ) of YM BPS monopoles (2.10):

$$f_1^{BPS} = [1 - \frac{r}{\epsilon \sinh r/\epsilon}] \Rightarrow f_1^{WY}, \quad (4.19)$$

when the mass of the Higgs field goes to infinity in the limit of the infinite spatial volume  $V \rightarrow \infty$ :

$$\frac{1}{\epsilon} = \frac{gm}{\sqrt{\lambda}} = \frac{g^2 \langle B^2 \rangle V}{4\pi} \rightarrow \infty. \quad (4.20)$$

Herewith BPS monopoles result the finite energy density [23]:

$$\int_{\epsilon}^{\infty} d^3x [B_i^a(\Phi_k)]^2 \equiv V \langle B^2 \rangle = 4\pi \frac{gm}{g^2 \sqrt{\lambda}} = \frac{4\pi}{g^2 \epsilon} \equiv \frac{1}{\alpha_s \epsilon}. \quad (4.21)$$

The infra-red cut-off parameter  $\epsilon$  disappears in the limit  $V \rightarrow \infty$ , i.e. when the mass of the Higgs field goes to infinity and Wu-Yang monopole turn, in a continuous wise, into BPS monopoles. In this case the BPS-regularization of Wu-Yang monopoles is similar to the *infrared regularization in QED* by introducing the so-called "photon mass"  $\lambda$  (see, e.g., [39], p. 413) that also violates the initial equations of motion <sup>4</sup>.

The vacuum energy density corresponding to (BPS, Wu-Yang) monopole solutions:

$$\sim \langle B^2 \rangle \equiv \frac{1}{\alpha_s \epsilon V} = \frac{4\pi m}{g \sqrt{\lambda} V}, \quad (4.22)$$

may be removed by the appropriate counter-term in the YM Lagrangian [23]:

$$\bar{\mathcal{L}} = \mathcal{L} - \frac{\langle B^2 \rangle}{2}. \quad (4.23)$$

In the existence of the nonzero vacuum "magnetic" tension  $\mathbf{B}$ , induced by the Bogomol'nyi equation (2.12), is the crucial distinction of the topological degeneration of fields in the Minkowski space from the topological degeneration of the classical vacuum in the instanton YM theory [3] in the Euclidean space  $E_4$  (where the  $F^{\mu\nu}(x) \rightarrow 0$  as  $x \rightarrow \infty$  normalization of the Euclidian YM tension tensor is a precondition of Eq. (4.5): see pp. 482- 483 in [13] <sup>5</sup>). The Bogol'nyi equation (2.12), compatible with the topology (2.7), ensures the existence of this nonzero vacuum "magnetic" tension.

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<sup>4</sup>It is necessary to note here that, in quantum-field theories (QFT), the transition to the limit  $V \rightarrow \infty$  is usually performed upon calculating physical observable values: scattering sections, probabilities of decays and so on; herewith these values would be normalized per time and volume units. Therefore all the specific features of the Minkowskian YM theory involving vacuum BPS monopoles, including the topological degeneration of initial data and vacuum "electric" monopoles (us discussed below), survive in any finite spatial volume.

<sup>5</sup>The author intends to give the detailed comparative analysis of the instanton Euclidian YM theory [3] and Minkowskian YM theory involving vacuum BPS (Wu-Yang) monopole modes in the series of next their works.

The problem is, indeed, *to formulate the Dirac quantization of weak perturbations of non-Abelian fields in the presence of non-perturbation monopole background*, taking account of the topological degeneration of all the initial data.

This will be the topic of the next subsection.

### 4.3 Dirac method and Gribov copies.

Instead of the artificial equations of the gauge-fixation method [40]:

$$F(A_\mu) = 0, \quad F(A_\mu^u) = M_F u \neq 0 \implies Z^{FP} = \int \prod_\mu DA_\mu \det M_F \delta(F(A)) e^{iW}, \quad (4.24)$$

we now repeat the Dirac *constraint-shell* formulation resolving the Gauss law constraint (3.5) with nonzero initial data <sup>6</sup>:

$$\partial_0 A_i^c = 0 \implies A_i^c(t, \mathbf{x}) = \Phi_i^{c(0)}(\mathbf{x}). \quad (4.25)$$

The vacuum *magneto-static* field  $\Phi_i^{c(0)}$  corresponds to the zero value of the winding number (4.2),  $X[\Phi_i^{c(0)}] = 0$ , and satisfies the classical equations everywhere excepting a small region near the origin of coordinates of the size

$$\epsilon \sim \frac{1}{\int d^3x B^2(\Phi)} \equiv \frac{1}{\langle B^2 \rangle V}, \quad (4.26)$$

(4.22), that disappears in the infinite volume limit.

The BPS monopole solutions for the Minkowskian YM vacuum in the zero topological sector, (2.10), is one of examples of resolving (4.25) the Gauss law constraint (3.5).

The second step is the consideration of the perturbation theory (4.12), in which the constraint (3.5) acquires the look

$$[D^2(\Phi^{(0)})]^{ac} A_{0c}^{(0)} = \partial_0 [D_i^{ac}(\Phi^{(0)}) A_c^i{}^{(0)}]. \quad (4.27)$$

Dirac proposed [26] to remove YM temporal components  $A_0$ .

The quantization of these non-dynamical degrees of freedom contradicts the quantum principles. More precisely, the non-dynamic status of  $A_0$  is not compatible with the quantization of this component due to its definite fixation (e.g., through the Gauss law constraint), while the appropriate zero canonical momentum

$$E_0 = \partial \mathcal{L} / \partial (\partial_0 A_0) = 0$$

contradicts the commutation relations and uncertainty principle [17].

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<sup>6</sup>Indeed (cf. Eqs. (15.12), (15.13) in [21]), the Gauss law constraint in the YM theory would contain the additional term, described *the total current* [25]. This is the sum of two items: the non-Abelian and fermionic currents. In the Minkowskian YM model here represented we treat this total current as a perturbation over the topologically degenerated BPS monopole vacuum [8, 25]. The consequences of this assumption we shall discuss in Sections 5 and 6.

Thus the constraint (4.27) acquires the form

$$\partial_0[D_i^{ac}(\Phi^{(0)})A_c^i{}^{(0)}] = 0. \quad (4.28)$$

We define the *constraint-shell gauge*

$$[D_i^{ac}(\Phi^{(0)})A_c^i{}^{(0)}] = 0 \quad (4.29)$$

as zero initial data for this constraint.

It is easy to see that the expression in the square brackets in (4.29) may be treated as *the equal to zero, in the initial time instant  $t = t_0$ , longitudinal component of a stationary YM field* (4.25). We shall denote it as  $A^{a\parallel}$ :

$$A^{a\parallel} \equiv [D_i^{ac}(\Phi^{(0)})A_c^i{}^{(0)}] = 0|_{t=0}. \quad (4.30)$$

Let us call the latter (*Cauchy*) condition as the *covariant Coulomb gauge*. Then the constraint (4.27) implies that also temporal derivatives of longitudinal fields equal to zero.

The topological degeneration of initial data (with  $n = 0$ ) implies *that not only the classical vacua, but also all the fields*, e.g.,

$$A_i^{(0)} = \Phi_i^{(0)} + \bar{A}_i^{(0)}, \quad (4.31)$$

in the transverse gauge (4.29) are degenerated:

$$\hat{A}_i^{(n)} = v^{(n)}(\mathbf{x})(\hat{A}_i^{(0)} + \partial_i)v^{(n)}(\mathbf{x})^{-1}, \quad v^{(n)}(\mathbf{x}) = \exp[n\Phi_0(\mathbf{x})]. \quad (4.32)$$

By analogy with (4.31), we may write down in any nonzero topological sector of the considered theory [8]:

$$\hat{A}_i^{(n)}(t, \mathbf{x}) = \Phi_i^{(n)}(\mathbf{x}) + \hat{\bar{A}}_i^{(n)}(t, \mathbf{x}). \quad (4.33)$$

We may explicitly write down gauge transformations leading to this topological degeneration [8]:

$$\hat{A}_k = v^{(n)}(\mathbf{x})T \exp \left\{ \int_{t_0}^t d\bar{t} \hat{A}_0(\bar{t}, \mathbf{x}) \right\} \left( \hat{A}_k^{(0)} + \partial_k \right) \left[ v^{(n)}(\mathbf{x})T \exp \left\{ \int_{t_0}^t d\bar{t} \hat{A}_0(\bar{t}, \mathbf{x}) \right\} \right]^{-1} \quad (4.34)$$

where the symbol  $T$  stands for the time ordering of the matrices under the exponent sign.

Thus in the initial time instant  $t_0$  the topological degeneration of initial data comes to "large" stationary matrices  $v^{(n)}(\mathbf{x})$ , depending on topological numbers  $n \neq 0$  and called the factors of the Gribov topological degeneration [24] or simply the *Gribov factors*. Thus we again come to the matrices  $v^T(\mathbf{x})$  in (3.24).

The linearized YM theory (3.7), (3.11) now prompts us how to generalize the exponential multipliers in (4.34) for the Minkowskian YM theory involving the physical BPS monopole vacuum to obtain transverse and physical Dirac variables (satisfying the

Coulomb gauge) in such a model. As far as we consider YM fields as perturbation excitations (multipoles) over the physical BPS monopole vacuum (see (4.12), (4.31), (4.33)), it is obviously that the exponential multipliers in (4.34) would explicitly depend on YM BPS monopole modes (2.10) (belonging to the zero topological sector), turning, in a continuous wise, into Wu-Yang monopoles (4.15) at the spatial infinity.

Thus we come to the following look for the exponential multipliers in (4.34) [8, 16, 17]:

$$U(t, \mathbf{x}) = v(\mathbf{x}) T \exp \left\{ \int_{t_0}^t \left[ \frac{1}{D^2(\Phi^{BPS})} \partial_0 D_k(\Phi^{BPS}) \hat{A}^k \right] d\bar{t} \right\}. \quad (4.35)$$

Following the work [8], we shall denote the exponential expression in (4.35) as  $U^D[A]$ ; this expression may be rewritten [8] as

$$U^D[A] = \exp \left\{ \frac{1}{D^2(\Phi^{BPS})} D_k(\Phi^{BPS}) \hat{A}^k \right\}. \quad (4.36)$$

We shall call the matrices  $U^D[A]$  as *Dirac "dressing"* of non-Abelian fields.<sup>7</sup>

The gauge transformations (4.34) are chosen to turn YM fields into physical (transverse) topological Dirac variables [23, 25].

Upon the transformation (4.3), the winding number functional (4.2) takes the look (see Eq. (181) in [17])

$$X[A_i^{(n)}] = X[A_i^{(0)}] + \mathcal{N}(n) + \frac{1}{8\pi^2} \int d^3x \epsilon^{ijk} \text{tr}[\partial_i(\hat{A}_j^{(0)} L_k^n)], \quad (4.37)$$

where the degree of the map  $\mathcal{N}(n) = n$  ( $n \in \mathbf{Z}$ ) is given by Eq. (4.4).

The constraint-shell gauge (4.29), (4.30) keeps its look in each topological class:

$$D_i^{ab}(\Phi_k^{(n)}) \bar{A}_b^{i(n)} = 0, \quad (4.38)$$

if the phase  $\hat{\Phi}_0(\mathbf{x})$  of Gribov topological factors  $v^{(n)}(\mathbf{x})$  satisfies *the equation of the Gribov ambiguity*

$$[D_i^2(\Phi_k^{(0)})]^{ab} \Phi_{(0)b} = 0; \quad (4.39)$$

this, in turn, implies the zero FP determinant  $\det \hat{\Delta}$  in (3.20).

On the face of it, this zero FP determinant may involve nontrivial Dirac dressing matrices (4.36) when we choose the Coulomb gauge (4.30). But in the initial time instant  $t = t_0$  the integral in (4.35) becomes zero; thus *in this time instant the Gribov ambiguity equation (4.39) does not affect the gauge transformations (4.34)*, i.e. the nature of topological Dirac variables and the Gribov topological degeneration.

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<sup>7</sup>In the work [8] the Dirac dressing matrices  $U^D[A]$  were represented in terms of Wu-Yang monopoles instead of BPS ones: (see, e.g., Eq. (C.1) in [8]). It is, of course, quite correct, as BPS monopoles turn, in a continuous wise, into Wu-Yang monopoles.

In other words, *in an arbitrary time instant  $t$  we may pick out a space-like surface  $\mathcal{H}(t)$  in the Minkowski space-time where the topological degeneration of initial YM data occurs.* This space-like surface is specified by the Gribov ambiguity equation (4.39).

Note that *the Gribov equation* (4.39), written down in terms of the Higgs isoscalar  $\Phi_{(0)b}$  (it is nothing else as a topologically trivial vacuum Higgs BPS monopole mode (2.9)), is the direct consequence of the Bogomol'nyj equation rewritten in the form (2.14) and Bianchi identity

$$\epsilon^{ijk}\nabla_i F_{jk}^b = 0.$$

Note that Eqs. (4.28), (4.30) may be treated as *Cauchy conditions* to the Gribov ambiguity equation (4.39).

Therefore to specify the space-like surface  $\mathcal{H}(t_0)$  where the topological degeneration of initial YM data occurs, we should solve the Cauchy task (4.39) with the initial conditions (4.28), (4.30), i.e. in the class of vacuum YM BPS monopoles (4.25) (and observable YM fields as perturbation excitations over this monopole vacuum possessing the same topological numbers that appropriate monopoles) satisfying the Coulomb gauge (4.30) when we resolve the Gauss law constraint (3.5) removing a la Dirac [26] temporal components of YM fields.

The existence of Gribov ambiguities in specifying transverse YM fields (satisfying the constraint-shell gauge (4.30), (4.38)) in each topological class of the Minkowskian YM theory is a purely non-Abelian effect (see, e.g., §T26 in [4]). Just the ambiguity in the choice of the Coulomb gauge (4.30) is given by Eq. (4.39) of the second order.

The Gribov ambiguity equation (4.39) permits a very interesting geometric interpretation.

We should recall, to begin with, that (see, e.g., §T22 in [4]) each gauge field  $A_\mu$ , as an element of the adjoint representation of the given Lie algebra, sets an element  $b_\gamma \in G$ , where  $G$  is the considered gauge Lie group. These elements  $b_\gamma$  are defined as

$$b_\gamma = P \exp\left(-\int_\gamma \mathbf{T} \cdot A_\mu dx^\mu\right), \quad (4.40)$$

with  $P$  being the symbol of the parallel transfer along the curve  $\gamma$  in a coordinate (for example, the Minkowski) space;  $\mathbf{T}$  being the matrices of the adjoint representation of the Lie algebra.

It is obviously that  $b_\gamma = b_{\gamma_1} b_{\gamma_2}$  as the end of the curve  $\Gamma_1$  coincides with the beginning of the curve  $\Gamma_2$  and the curve  $\Gamma$  is formed from these curves ( $\Gamma = \Gamma_1 \cup \Gamma_2$ ). Thus the group operation (the multiplication) is associative; also always there exists the unit element and element inverse to the given one. This follows from the usual features of curves and the exponential function.

Thus elements  $b_\gamma$  form a subgroup in the given gauge group  $G$ . Moreover, it is easy to see that this subgroup coincides with  $G$  and may be treated as its specific (adjoint) representation.

We see that elements  $b_\gamma$  are specified over the set of external 1-forms in the given Lie algebra. The cohomologies classes of these external 1-forms are the elements of the cohomologies group  $H^1$  (see, e.g., §T7 in [4]).

The Pontryagin formula for a degree of a map (see, e.g., Lecture 26 in [41]):

$$\int_X f^* \omega = \deg f \in \mathbf{Z} \quad (4.41)$$

(where the map  $f : X \rightarrow Y$  is smooth and maps the compact space  $X$  into the compact space  $Y$ : it is the so-called *eigen map*;  $f^*$  is the conjugate homomorphism of the appropriate cohomologies groups:  $H_X^1 \rightarrow H_Y^1$ , induced by the map  $f$ ), sets a *one-to-one correspondence between the homotopies and cohomologies groups* in the considered theory <sup>8</sup>.

In particular, topological charges  $n = 0$  correspond to *exact* 1-forms, i.e. to those that may be represented as differentials,  $d\sigma$ , of 1-forms  $\sigma$ . Due to the *Poincare lemma*,

$$d \cdot d\sigma = 0,$$

i.e. *each exact form is closed*.

A scalar field  $\Phi_{(0)b}$  entering the Gribov equation (4.39) belongs to the zero topological sector:  $n = 0$ . This zero charge, through the *smooth* (and without additional singularity sources) Bogomol'nyj equation (2.14) [4] <sup>9</sup>, is told to the magnetic tension tensor  $F_{jk}^b$  (i.e. to appropriate 2-forms) and to appropriate YM fields (1-forms).

Let us now consider those curves  $\Gamma$  that begin and finish at the same point  $x_1$  of the Minkowski space. Such closed curves are called *the one-dimensional cycles*, and the appropriate elements  $b_\gamma$  may be written down as cyclic integrals:

$$b_\gamma = P \exp(- \oint_\Sigma \mathbf{T} \cdot A_\mu dx^\mu), \quad (4.42)$$

over one-dimensional cycles  $\Sigma$ . According to the *De Rham theorem* (see p. 276 in [4]), if an external form  $\omega$  is exact, its integral over *each* cycle specified over the considered manifold  $M$  is equal to zero (this also confirms Eq. (4.41) for topological charges  $n = 0$ ).

The said implies that the integral in (4.42) is equal to zero ( $b_\gamma = 1$ ) *for each exact form* (corresponding to the zero topological charge due to (4.41)).

The elements (4.42) form *the holonomies subgroup*  $H$  in the given gauge group. In the considered Minkowskian YM model [8, 17, 23, 25] it is the residual  $U(1)$  gauge symmetry group, that is embedded in a nontrivial wise in the initial gauge symmetry group  $SU(2)$ . Herewith  $U(1)$  plays the role of the invariant subgroup in  $SU(2)$ ; this, in turns, involves the nontrivial embedding of the holonomies group  $H$  "belonging" to the  $U(1)$  gauge symmetry into the holonomies group  $H'$  "belonging" to the  $SU(2)$  gauge symmetry.

Those of elements (4.42) that correspond to exact 1-forms at the just described embedding form, in turn, the so-called *restricted holonomies subgroup* in the residual gauge symmetry group  $U(1)$  <sup>10</sup>. We shall denote it as  $\Phi^0$  henceforth.

<sup>8</sup>We should apply Eq. (4.41) to the topology (2.7) in the considered Minkowskian YM model.

<sup>9</sup>It is smooth, in effect, out of the  $\epsilon$ -neighbourhood of the origin of coordinates, where the value  $\epsilon$  was specified by Eq. (4.20) as the typical size of BPS monopoles [4, 8, 23, 25]. At the origin of coordinates the vacuum "magnetic" tension  $\mathbf{B}$  becomes infinite and diverges as  $r^{-2}$  [6].

<sup>10</sup>In some modern physical literature, e.g. in the papers [43], this group refers to as *the trivial holonomies group*.

It is easy to see that in the Minkowskian YM model [8, 17, 23, 25] the restricted holonomies group  $\Phi^0$  is isomorphic to the "small" subgroup  $U_0$  in the residual gauge symmetry group  $U(1)$  represented by Gribov topological multipliers  $v^{(n)}(\mathbf{x})$ , (4.32).

Indeed, only "small" Gribov multipliers form the complete group  $U_0$  ( $\pi_0(U_0) = 0$ ;  $\pi_1(U_0) = 0$ ), while each set of "large" Gribov multipliers with the fixed topological number  $n \in \mathbf{Z}$  is only a *monoid*; the element inverse to the given one in the said "large" set with  $n$  always belongs to the set with  $-n$  ( $n \neq 0$ ):

$$v^{(-n)}(\mathbf{x}) = [v^{(n)}(\mathbf{x})]^{-1}.$$

One may speak that the unit element of the holonomies group  $H$  is *degenerated* with respect to all the exact forms corresponding to zero topological charges. In our case of vacuum Higgs fields  $\Phi_{(0)b}$  just these fields specify the class of exact forms and restricted holonomies group  $\Phi^0$ .

As it was shown in the monographs [4], when  $A_\mu$  and  $A'_\mu$  are two gauge fields the gauge equivalence between which is realized through a function  $g(x) \in G$ , then

$$b'_\gamma = g(x_1)b_\gamma g(x_1)^{-1} \quad (4.43)$$

as the curve  $\Gamma$  begins and ends in the point  $x_1$  (*the holonomies elements of two gauge equivalent fields are conjugate*).

Let  $g(x)$  have the spatial asymptotic  $g(x) \rightarrow 1$  as  $x \rightarrow \infty$  (indeed,  $g(x)$  would have such asymptotic already on distances  $\sim 1$  fm.; this is associated with the needs of the infrared quark confinement as we shall see below). Let also  $b_\gamma$  and  $b'_\gamma$  be elements of  $H$  constructed by external forms belonging to a one class of cohomologies. In conclusion, the gauge fields  $A_\mu$  and  $A'_\mu$ , forming the elements  $b_\gamma$  and  $b'_\gamma$  respectively (these fields "belong" to the  $U(1) \rightarrow SU(2)$  embedding) and connected by the gauge transformation  $g(x) \rightarrow 1$ , *are taken in the Coulomb gauge* (4.38).

Each such class is obtained from the zero class of exact forms *as its Gribov copy* (thus we also "copy" the Gribov ambiguity equation (4.39) written down in terms of Higgs fields with zero topological charges).

As  $g(x) \rightarrow 1$ ,  $x \rightarrow \infty$ , we may rewrite (4.43) as

$$b'_\gamma = b_\gamma \cdot 1. \quad (4.44)$$

The latter equality reflects the structure of the cohomologies group  $H^1$ : *two 1-forms belonging to one class of cohomologies are equivalent to within an exact form* (see §T6 in [4]).<sup>11</sup> In terms of the holonomies group  $H$  this, in turn, implies that two elements of  $H$

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<sup>11</sup>We may directly make sure that always there exists the infinite set of transverse YM fields in each cohomological class. As two 1-forms  $\omega_1 \equiv A_\mu d^\mu$  and  $\omega'_1 \equiv A'_\mu d^\mu$ , involving the transverse YM fields  $A_\mu$  and  $A'_\mu$  respectively, belonging to the fixed cohomologies class differ on the exact 1-form  $d\sigma$ :  $\omega_1 - \omega'_1 = d\sigma$ , we may approximately write:

$$\partial_\mu(\omega_1 - \omega'_1) = \partial_\mu d\sigma = 0,$$

as far as  $d \cdot d\sigma = 0$  due to the Poincare lemma.

corresponding to the 1-forms belonging to a one class of cohomologies are *equivalent to within an element of the restricted holonomies group*  $\Phi^0$ .

Thus the Gribov ambiguity equation (4.39) and Gribov gauge transformations (4.34) correctly describe the nontrivial chorological structure of the transverse vacuum YM fields satisfying the Coulomb gauge (4.38) and involving the non-Abelian gauge group  $SU(2)$  then broken down spontaneously to the  $U(1)$  gauge group.

Note that specifying the above cohomological structure of transverse YM fields, we herewith solve, in fact, the Cauchy task (4.39), (4.28), (4.30) (*expanded on all the cohomologies classes due to the Gribov topological mechanism* (4.34) and *Pontryagin theory* (4.41)), i.e. define, in the given initial instant  $t_0$ , the space-like surface  $\mathcal{H}(t_0)$  in the Minkowski space-time where the Gribov topological degeneration of initial data occurs.

One may show [23, 25] that the Gribov equation (4.39) together with the topological condition

$$X[\Phi^{(n)}] = n \quad (4.45)$$

are compatible with the unique solution to the classical equations. *They just result Wu-Yang monopoles  $\Phi_k^{(n)}$  considered above.*

The nontrivial solution to the equation for the Gribov phase in this case is well known [8, 17]:

$$\hat{\Phi}_0(r) = -i\pi \frac{\tau^a x_a}{r} f_{01}^{BPS}(r), \quad f_{01}^{BPS}(r) = \left[ \frac{1}{\tanh(r/\epsilon)} - \frac{\epsilon}{r} \right]. \quad (4.46)$$

It is just a  $U(1) \rightarrow SU(2)$  isoscalar made of Higgs vacuum BPS monopoles (2.9).

As a definite linear combination of Higgs vacuum BPS monopole modes (2.9), the Gribov phase  $\hat{\Phi}_0$  satisfies the Gribov ambiguity equation (4.39) [8, 17].

As a result, instead of the topological degenerated classical vacuum inherent in the instanton theory [3] (in the physically unattainable region), we get a topologically degenerated Wu-Yang (BPS) monopole:

$$\hat{\Phi}_i^{(n)} := v^{(n)}(\mathbf{x}) [\hat{\Phi}_i^{(0)} + \partial_i] v^{(n)}(\mathbf{x})^{-1}, \quad v^{(n)}(\mathbf{x}) = \exp[n\hat{\Phi}_0(\mathbf{x})], \quad (4.47)$$

and topological degenerated multipoles:

$$\hat{A}^{(n)} := v^{(n)}(\mathbf{x}) \hat{A}^{(0)} v^{(n)}(\mathbf{x})^{-1}. \quad (4.48)$$

As regards Higgs vacuum BPS monopoles, they are also topologically degenerated and herewith in the same wise that YM multipoles [34]:

$$\Phi_{(n)b} = v^{(n)}(\mathbf{x}) \Phi_{(0)b} v^{(n)}(\mathbf{x})^{-1}. \quad (4.49)$$

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Moreover, one may be proved that always takes place the actual *duplication* of the infinite number of transverse YM fields in each cohomological class. It is the direct consequence of the Gribov ambiguity equation (4.39) and nontrivial  $U(1) \rightarrow SU(2)$  embedding. Applying the arguments similar to those in the work [44], we may always choose two infinite close to each other transverse YM fields (in each topological sector of the Minkowskian YM model [8, 17, 23, 25]) in such a wise that one of these fields belongs to the Lee algebra of  $SU(2)$ , while another one belongs to the Lee algebra of  $U(1)$ .



Let us now examine the behaviour of the Gribov phase (4.46) at the origin of coordinates and at the spatial infinity.

The BPS ansatz  $f_{01}^{BPS}(r) = f_0^{BPS}(r) \epsilon$  in this formula becomes 1 as  $r \rightarrow \infty$  [17, 25], while at the origin of coordinates it goes to zero.

It turns out that it is a sign of a good physics.

Really, in this case Gribov exponential multipliers

$$v^{(n)}(\mathbf{x}) = \exp[n\hat{\Phi}_0(\mathbf{x})]$$

become 1 at the origin of coordinates.

On the other hand, at the spatial infinity the Gribov factors  $v^{(n)}(\mathbf{x})$  would have the same asymptotic, to ensure the topological and quark confinements in the infrared region of the momentum space [8, 17, 23, 30, 31, 32, 45] via the complete destructive interference of "large" Gribov multipliers  $v^{(n)}(\mathbf{x})$  ( $n \neq 0$ ) at the spatial infinity.

The features of the infrared topological (and quark) confinement [17, 23, 30, 31, 32, 45] will be us discussed below. But now we would like to consider the conditions that is necessary to impose onto ("large") Gribov multipliers  $v^{(n)}(\mathbf{x})$  to achieve their spatial asymptotic

$$v^{(n)}(\mathbf{x}) \rightarrow 1, \quad \text{as } |\mathbf{x}| \rightarrow \infty, \quad (4.50)$$

providing the infrared topological confinement.

Let us slightly modify Eq. (4.46) for the Gribov phase in comparison with that in the works [8, 17, 25]. The paper [45] prompts us how to do this.

So, we rewrite Gribov exponential multipliers  $v^{(n)}(\mathbf{x})$  as [45]

$$v^{(n)}(\mathbf{x}) = \exp(\hat{\lambda}_{n,\phi_i}(\mathbf{x})), \quad (4.51)$$

where

$$\hat{\lambda}_{n,\phi_i}(\mathbf{x}) \equiv 2i\tau^a \Omega_{ab}(\phi_i) \frac{x^b}{r} f_{01}^{BPS}(r) \pi n \quad (4.52)$$

and

$$\begin{aligned} (\tau^a)_\beta^\alpha \Omega_{ab}(\phi_i) &= (u(\phi_i))_\gamma^\alpha (\tau^a)_\delta^\gamma (u^{-1}(\phi_i))_\beta^\delta, \\ (u(\phi_i))_\beta^\alpha &= (e^{i\tau_1\phi_1/2})_\gamma^\alpha (e^{i\tau_2\phi_2/2})_\delta^\gamma (e^{i\tau_3\phi_3/2})_\beta^\delta. \end{aligned} \quad (4.53)$$

Here  $\phi_i$  ( $i = 1, 2, 3$ ) are three Euler angles, fixing the position of the coordinate system in the  $U(1) \rightarrow SU(2)$  group space.

To obtain the necessary asymptotic (4.50) at the spatial infinity, we should impose the appropriate conditions onto the Gribov phase in the modified Eq. (4.51). More precisely, such conditions may be imposed onto Euler angles  $\phi_i$ .

*Gribov topological factors  $v^{(n)}(\mathbf{x})$  become 1 at the spatial infinity as*

$$\tau_i \phi_i = 4\pi n, \quad n \in \mathbf{Z}. \quad (4.54)$$

#### 4.4 Topological dynamics and chromo-electric monopole.

Gribov copies of YM fields belonging to the zero topological sector of the Minkowskian theory involving vacuum BPS monopoles are an evidence of the zero mode in the left-hand side of the both constraints (3.5), (4.27):

$$[D_i^2(\Phi^{(0)})]^{ac}A_{0c} = 0. \quad (4.55)$$

The nontrivial solution to this equation,

$$A_0^c(t, \mathbf{x}) = \dot{N}(t)\Phi_0^c(\mathbf{x}) \quad (4.56)$$

(it is, indeed, the solution of the Gauss law constraint (3.5) additional [16] to the trivial one,  $A_0 = 0$ , obtained at the Dirac removal [26] of temporal YM components), may be removed from the local equations of motion by the gauge transformation (3.25) (a la Dirac in 1927 y. [26]) to convert YM fields *into topological Dirac variables*:

$$\hat{A}_i^{(N)} = \exp[N(t)\hat{\Phi}_0(\mathbf{x})][\hat{A}_i^{(0)} + \partial_i] \exp[-N(t)\hat{\Phi}_0(\mathbf{x})]. \quad (4.57)$$

But the solution (4.56) cannot be removed from the constraint-shell action

$$W^* = \int dt \dot{N}^2(t)I/2 + \dots$$

and from the winding number functional

$$X[A^{(N)}] = N(t) + X[A^{(0)}]$$

(the latter equality implies that local YM degrees of freedom  $A^{(N)}$  are completely separated from  $N(t)$  [8]).

Finally, we get the Feynman path integral

$$Z_F = \int DN \prod_{i,c} [DE_i^{c(0)} DA_c^{i(0)}] e^{iW^*}, \quad (4.58)$$

that contains the additional topological variable  $N(t)$ .

We shall consider deriving the integral (4.58) in the next sections.

In the lowest order of the perturbation theory the constraint (4.55) permits the solution (4.56), implying a vacuum **"electric" monopole**

$$F_{i0}^b = \dot{N}(t)D_i^{bc}(\Phi_k^{(0)})\Phi_{(0)c}(\mathbf{x}). \quad (4.59)$$

We call the just appeared variable  $N(t)$  *the winding number variable*. It is specified with the aid of the vacuum Chern-Simons functional (in the zero topological sector), equal to the difference of the *in* and *out* values of this variable:

$$\begin{aligned} \nu[A_0, \Phi^{(0)}] &= \frac{g^2}{16\pi^2} \int_{t_{in}}^{t_{out}} dt \int d^3x F_{\mu\nu}^a \tilde{F}^{a\mu\nu} = \frac{\alpha_s}{2\pi} \int d^3x F_{i0}^b B_i^b(\Phi^{(0)}) [N(t_{out}) - N(t_{in})] \\ &= N(t_{out}) - N(t_{in}). \end{aligned} \quad (4.60)$$

The winding number functional admits its generalization *to noninteger degrees of the map* [36]:

$$X[\Phi^{(N)}] \neq n \quad (n \in \mathbf{Z}), \quad (\hat{\Phi}^{(N)} = e^{N\hat{\Phi}_0}[\hat{\Phi}_i^{(0)} + \partial_i]e^{-N\hat{\Phi}_0}). \quad (4.61)$$

We may identify the global variable  $N(t)$  with the winding number degree of freedom in the Minkowski space associated with the *free rotator action*

$$W_N = \int d^4x \frac{1}{2} (F_{0i}^c)^2 = \int dt \frac{\dot{N}^2 I}{2}, \quad (4.62)$$

involving the rotary momentum (cf. (4.6) in [23]):

$$I = \int_V d^3x (D_i^{ac}(\Phi_k)\Phi_{(0)c})^2 = \frac{4\pi^2\epsilon}{\alpha_s} = \frac{4\pi^2}{\alpha_s^2} \frac{1}{V \langle B^2 \rangle}, \quad (4.63)$$

does not contribute to the local equations of motion. This free rotator action disappears in the limit  $V \rightarrow \infty$  ( $\epsilon \rightarrow 0$ ).

The influence of the infinite mass of the Higgs field,  $m/\sqrt{\lambda}$ , onto the free rotator action (4.62) reduces, at the spatial infinity ( $V \rightarrow \infty$ ), in fact to zero, as we learn this from (4.63).

The rotation action (4.62) describes the collective solid potential rotation of the Minkowskian (YM-Higgs) vacuum [8, 17, 23, 25] (the latter one is, indeed, a kind of superfluid Bose condensate [5], alike to the superfluid component in a helium II specimen).

Our reasoning about  $\langle B^2 \rangle \neq 0$  (this value is induced by the Bogomol'nyi equation (2.12) and characterizes the superfluid (YM-Higgs) vacuum), confirms that the action (4.62), depending on the *collective topological variable*  $N(t)$  via its temporal derivative  $\dot{N}(t)$  and describing the collective solid potential rotation of the Minkowskian (YM-Higgs) vacuum [8, 17, 23, 25] (in fact, the action (4.62) is induced by vacuum Higgs BPS monopole modes (2.9)), is a purely Minkowskian effect.

Now, taking account of evaluating the magnetic energy (4.21), we may write down the action of the of the Minkowskian YM theory [8, 17, 23, 25] in the lowest order of the perturbations theory, describing only the (physical) vacuum of this theory.

This action [8] contains the both kinds of vacuum BPS monopoles, the "electric" and the "magnetic" ones:

$$W_{\mathcal{Z}}[N, \Phi^{0BPS}] = \int dt d^3x \frac{1}{2} \{ [F_{i0}^b]^2 - [B_i^b(\Phi^{0BPS})]^2 \} = \int dt \frac{1}{2} \{ I \dot{N}^2 - \frac{4\pi}{g^2\epsilon} \}. \quad (4.64)$$

The topological degeneration of all the fields reduces to the degeneration of the global topological variable  $N(t)$  with respect to the shift of this variable on integers:

$$N \implies N + n; \quad n = \pm 1, \pm 2, \dots; \quad 0 \leq N(t) \leq 1.$$

Thus the topological variable  $N(t)$  specifies the free rotator (4.62) involving the instanton-type wave function (4.9) of the topological motion in the Minkowski space-time:

$$\Psi_N = \exp(iP_N N), \quad P_N = \dot{N}I = 2\pi k + \theta, \quad (4.65)$$

with  $k$  being the *number of the Brillouin zone* and  $\theta$  being the  $\theta$ -angle (or the *Bloch quasi-momentum*) [8, 16, 17].

The action (4.64) of the Minkowskian YM theory [8, 17, 23, 25] in the lowest order of the perturbations theory induces the appropriate vacuum Hamiltonian (written down in terms of the canonical momentum  $P_N = \dot{N}I$ ) [8]:

$$H = \frac{2\pi}{g^2\epsilon} [P_N^2 (\frac{g^2}{8\pi^2})^2 + 1]. \quad (4.66)$$

We see that this Hamiltonian, depending on  $P_N^2$ , is explicitly *Poincare invariant* (in particular, CP invariant), unlike the  $\theta$ -term in the effective Lagrangian of the instanton YM theory [3].

Thus the well-known CP-problem [13] may be solved in the Minkowskian YM theory [8, 17, 23, 25] involving vacuum "electric" monopoles (4.59).

In contrast to the instanton wave function (4.9), inherent in the Euclidian YM theory [3] and involving *the purely imaginary energy-momentum spectrum* [8, 17, 34, 46], the spectrum of the topological momentum  $P_N$  in the Minkowskian YM theory [8, 17, 23, 25] *is real and belongs to the physical values*.

Finally, Eqs. (4.62), (4.65) specify *the countable spectrum* of the global "electric" tension (4.59):

$$F_{i0}^a = \dot{N}(t) (D_i(\Phi_k^0)\Phi_0)^a = P_N \frac{\alpha_s}{\pi^2\epsilon} B_i^a(\Phi_0) = |2\pi k + \theta| \frac{\alpha_s}{\pi^2\epsilon} B_i^a(\Phi_0). \quad (4.67)$$

It is an analogue of the Coleman spectrum of the electric tension in the  $QED_{(1+1)}$  [47]:

$$G_{10} = \dot{N} \frac{2\pi}{e} = e \left( \frac{\theta}{2\pi} + k \right). \quad (4.68)$$

The application of the Dirac quantization to one-dimensional electrodynamics  $QED_{(1+1)}$  in the paper [48] demonstrates the universality of (topological) Dirac variables and their adequacy to the description of topological dynamics in terms of a nontrivial homotopies group.

## 5 Zero mode of Gauss law constraint.

### 5.1 Dirac variables and zero mode of Gauss law constraint.

The Minkowskian YM constraint-shell theory [8, 17, 23, 25] is obtained at explicit resolving the Gauss law constraint (3.5), and our next step is associated with the initial action on the surface of these solutions:

$$W^* = W[A\mu] \Big|_{\frac{\delta W}{\delta A_0}=0}. \quad (5.1)$$

The results of similar resolving in QED are *electrostatic and Coulomb-like atoms*.

In the non-Abelian case the topological degeneration in the form of Gribov copies (4.32) implies that the general solution to the Gauss law constraint (3.5) contains the zero mode  $\mathcal{Z}$  as a peculiar indicator of the Coulomb gauge (4.30).

The general solution to the heterogeneous equation (3.5) may be represented [8, 16] as the sum of the zero-mode solution  $\mathcal{Z}^a$  to the homogeneous equation

$$(D^2(A))^{ab}\mathcal{Z}_b = 0 \quad (5.2)$$

and a particular solution,  $\tilde{A}_0^a$ , to the heterogeneous one, that is

$$A_0^a = \mathcal{Z}^a + \tilde{A}_0^a. \quad (5.3)$$

On the other hand, the zero-mode  $\mathcal{Z}^a$ , may be represented, at the spatial infinity, in the form of the sum of the product of the new topological variable  $\dot{N}(t)$  onto the Gribov phase  $\hat{\Phi}_{(0)}(\mathbf{x})$ , (4.46), and weak multipole corrections:

$$\hat{\mathcal{Z}}(t, \mathbf{x})|_{\text{asymptotic}} = \dot{N}(t)\hat{\Phi}_{(0)}(\mathbf{x}) + O\left(\frac{1}{r^{l+1}}\right), \quad (l > 1). \quad (5.4)$$

In this case the single one-parametric variable  $N(t)$  reproduces the topological degeneration of all the YM field as the topological Dirac variables are specified by the gauge transformations

$$0 = U_{\mathcal{Z}}(\hat{\mathcal{Z}} + \partial_0)U_{\mathcal{Z}}^{-1} \quad (5.5)$$

$$\hat{A}_i^* = U_{\mathcal{Z}}(\hat{A}_i^0 + \partial_i)U_{\mathcal{Z}}^{-1}, \quad \hat{A}_i^{(0)} = \Phi_i^{(0)} + \bar{A}_i^{(0)}, \quad (5.6)$$

where the spatial asymptotic of  $U_{\mathcal{Z}}$  is

$$U_{\mathcal{Z}} = T \exp\left[\int^t dt' \hat{\mathcal{Z}}(t', \mathbf{x})\right]|_{\text{asymptotic}} = \exp[N(t)\hat{\Phi}_{(0)}(\mathbf{x})]. \quad (5.7)$$

The topological degeneration of all the fields reduces in this case to the degeneration of only one global topological variable  $N(t)$  with respect to the shift of this variable on integers: ( $N \Rightarrow N + n, n = \pm 1, \pm 2, \dots$ ).

## 5.2 Constraining with zero mode.

Let us specify *the equivalent unconstrained system* for the Minkowskian YM theory [8, 17, 23, 25] in the monopole class of functions in the presence of the zero mode  $\mathcal{Z}^b$  of the Gauss law constraint (3.5), (5.2):

$$A_0^a = \mathcal{Z}^a + \tilde{A}_0^a; \quad F_{0k}^a = -D_k^{ab}(A)\mathcal{Z}_b + \tilde{F}_{0k}^a \quad ((D^2(A))^{ab}\mathcal{Z}_b = 0). \quad (5.8)$$

To get the constraint-shell action:

$$W_{YM}(\text{constraint}) = \mathcal{W}_{YM}[\mathcal{Z}] + \tilde{W}_{YM}[\tilde{F}], \quad (5.9)$$

we use the obvious decomposition:

$$F^2 = (-D\mathcal{Z} + \tilde{F})^2 = (D\mathcal{Z})^2 - 2\tilde{F}D\mathcal{Z} + \tilde{F}^2 = \partial(\mathcal{Z} \cdot D\mathcal{Z}) - 2\partial(\mathcal{Z}\tilde{F}) + (\tilde{F})^2. \quad (5.10)$$

The latter relation is true due to the Bianchi identity  $D\tilde{F} = 0$ , the Gauss law constraint equation  $D^2\mathcal{Z} = 0$  and the explicit expression for the derivative  $D\mathcal{Z}$ :

$$D\mathcal{Z} = (\partial\mathcal{Z} + g\mathbf{A} \times \mathcal{Z}).$$

This shows that the zero mode part,  $\mathcal{W}_{YM}[\mathcal{Z}]$ , of the constraint-shell action (5.9) is, indeed, the sum of two integrals:

$$\mathcal{W}_{YM}[\mathcal{Z}] = \int dt \int d^3x \left[ \frac{1}{2} \partial_i (\mathcal{Z}^a D_{ab}^i(A) \mathcal{Z}^b) - \partial_i (\tilde{F}_{0i}^a \mathcal{Z}_a) \right] = \mathcal{W}^0 + \mathcal{W}', \quad (5.11)$$

where the first term,  $\mathcal{W}^0$ , is the *action of a free rotator*, (4.62), and the second one,  $\mathcal{W}'$ , describes *the coupling of the zero-mode to local excitations*.

These terms are specified by the asymptotic of fields  $(\mathcal{Z}^a, A_i^a)$  at the spatial infinity. We shall denote them as  $\dot{N}(t)\Phi_{(0)}^a(\mathbf{x})$  and  $\Phi_i^a(\mathbf{x})$  respectively. The fluctuations  $\tilde{F}_{0i}^a$  belong to the class of multipoles.

As long as the surface integral over the monopole-multipole couplings vanishes (due to the Gauss-Ostrogradsky theorem and spatial asymptotic (5.4) for the zero mode solution), the fluctuation part of the second term drops out: there are no contributions from the interference between the "electric" monopole and perturbation excitations: multipoles and Higgs "electric" excitations, in the constraint-shell action (5.9).

Thus [8] scalar BPS monopoles disappear, in effect, from the excitations spectrum in the infinite spatial volume limit  $V \rightarrow \infty$ .

Substituting the solution with the asymptotic (5.4) into the first term of Eq. (5.11) implies the free rotator action (4.62).

The action for the equivalent unconstrained system of local excitations (cf. Eq. (21) in [17]):

$$\tilde{W}_{YM}[\tilde{F}] = \int d^4x \left\{ E_k^a \dot{A}_a^{k(0)} - \frac{1}{2} \{ E_k^2 + B_k^2(A^{(0)}) + [D_k^{ab}(\Phi^{(0)}) \tilde{\sigma}_b]^2 \} \right\}, \quad (5.12)$$

is obtained in terms of variables with the zero degree of the map:

$$\hat{\tilde{F}}_{0k} = U_{\mathcal{Z}} \tilde{F}_{0k}^{(0)} U_{\mathcal{Z}}^{-1}, \quad \hat{A}_i = U_{\mathcal{Z}} (\hat{A}_i^{(0)} + \partial_i) U_{\mathcal{Z}}^{-1}, \quad \hat{A}_i^{(0)}(t, \mathbf{x}) = \Phi_i^{(0)}(t, \mathbf{x}) + \hat{\tilde{A}}^{(0)}(t, \mathbf{x}), \quad (5.13)$$

by decomposing the "electric" component of the YM tension tensor  $F_{0i}^{(0)}$  into their transverse:  $E_i^a$ , and longitudinal:

$$F_{0i}^{aL} = -D_i^{ab}(\Phi^{(0)}) \tilde{\sigma}_b,$$

parts; so

$$F_{0i}^{a(0)} = E_i^a - D_i^{ab}(\Phi^{(0)}) \tilde{\sigma}_b. \quad (5.14)$$

Here the function  $\tilde{\sigma}^b$  is specified by the Gauss law equation (3.5) rewritten in terms of Dirac variables:

$$((D^2(\Phi^{(0)}))^{ab} + g\epsilon^{adc}\tilde{A}_{id}^{(0)}D_c^{ib}(\Phi^{(0)}))\tilde{\sigma}_b = -g\epsilon^{abc}\tilde{A}_{ib}^{(0)}E_c^i \quad (5.15)$$

(we recommend our readers the monograph [22], p. 88, where Eqs. (5.14), (5.15) were derived in the Hamiltonian formalism of the YM theory).

The transverse "electric" tension tensor  $E_i^a$  appears in the considered Minkowskian YM theory. We may write down it in terms of the zero mode  $\mathcal{Z}^a$  of the Gauss law constraint (5.2) [23]:

$$E_i^a = D_i^{ac}(A)A_{0ac}, \quad (5.16)$$

with  $A_0^a$  given in (5.3). Due to (5.3), the "electric" tension tensor  $E_i^a$  may be, in turn, decomposed into the zero mode part and perturbations one,  $\tilde{E}_j^c$ :

$$E_i^a = D_i^{ac}\mathcal{Z}_c + \tilde{E}_i^a; \quad (5.17)$$

the latter item enters the unconstrained excitation action (5.12).

In order to construct the consequent QCD in the Minkowski space, we should incorporate fermions (quarks) in that theory. We also consider them as perturbation excitations over the Minkowskian physical (YM, gluonic) vacuum [8, 17, 23, 25]. (we shall treat the latter one as the ground level of the perturbations theory). Thus we may supplement the unconstrained excitation action (5.12) by the fermionic contribution [23, 25]:

$$\tilde{W}[\Psi] = \int d^4x \bar{\psi}[i\gamma^\mu(\partial_\mu + \hat{A}_\mu) - m]\psi. \quad (5.18)$$

When we then introduce the current  $j$  of independent non-Abelian variables [8]:

$$j_0^a = g\epsilon^{abc}[A_{ib}^{*a(n)} - \Phi_{ib}^{a(n)}]\tilde{E}_c^i, \quad (5.19)$$

the fermionic term (5.18) allows us to consider the so-called *total current*: the sum of the current  $j$ , (5.19), of independent non-Abelian variables, and a fermionic current [25]:

$$j_{tot\ 0}^b = g\epsilon^{abc}\tilde{A}_i^a\tilde{E}_i^c + j_0^{fb}, \quad (5.20)$$

with  $j_0^{fb}$  being a fermionic current.

Then Eq. (5.15) may be rewritten as

$$D_i^{cd}(A)D_{db}^i(\Phi^{(0)})\tilde{\sigma}^b = j_0^c. \quad (5.21)$$

The latter equation is wholly determined by the perturbation excitations over the zero mode  $\hat{\mathcal{Z}}$ , (5.4).

Due to its manifest gauge invariance, the dependence of the action for local excitations on the zero mode disappears, and we get the ordinary generalization of the covariant Coulomb gauge [19, 20, 21] in the presence of vacuum Wu-Yang (BPS) monopoles.

## 6 Rising potential induced by Wu-Yang monopoles.

Now we are able to calculate the Green function of the Gauss constraint equation (5.21), coinciding mathematically with the Gribov equation (4.39) (see [23], §4.C):

$$D^2((\Phi^{(0)})^{ab}(\mathbf{x})G_b^c(\mathbf{x}, \mathbf{y}) = \delta^{ac}\delta^3(x - y), \quad (6.1)$$

that sets, due to (5.20), (5.21), the potential of the current-current instantaneous interaction:

$$-\frac{1}{2} \int_{V_0} d^3x d^3y j_{tot,0}^b(\mathbf{x}) G_{bc}(\mathbf{x}, \mathbf{y}) j_{tot,0}^c(\mathbf{y}). \quad (6.2)$$

Note [25] that Eq. (6.1) allows us to find the "longitudinal" function  $\tilde{\sigma}_b$  as an expansion by powers of  $g$ :

$$\tilde{\sigma}^b(t, \mathbf{x}) = - \int d^3y G^{bc}(\mathbf{x}, \mathbf{y}) j_{tot,0}^c(t, \mathbf{y}) - \int d^3y d^3z G^{bc}(\mathbf{x}, \mathbf{y}) g f^{cde} \tilde{A}_k^d(t, \mathbf{y}) G^{ef}(\mathbf{y}, \mathbf{z}) j_{tot,0}^f(t, \mathbf{z}) - \dots, \quad (6.3)$$

by analogy with the standard perturbations theory [49].

In the presence of a vacuum Wu-Yang monopole we get

$$D^2((\Phi^{(0)})^{ab}(\mathbf{x}) = \delta^{ab} \Delta - \frac{n^a n^b + \delta^{ab}}{r^2} + 2\left(\frac{n^a}{r} \partial^b - \frac{n^b}{r} \partial^a\right), \quad (6.4)$$

with

$$n_a(x) = x_a/r; \quad r = |\mathbf{x}|.$$

Let us decompose  $G^{ab}$  into the complete set of orthogonal vectors in the colour space:

$$G^{ab}(\mathbf{x}, \mathbf{y}) = [n^a(x) n^b(y) V_0(z) + \sum_{\alpha=1,2} e_\alpha^a(x) e^{b\alpha}(y) V_1(z)]; \quad (z = |\mathbf{x} - \mathbf{y}|). \quad (6.5)$$

Substituting the latter expression into Eq. (6.1), we get *the Euler equation* (see [50], Eq. (2.160)):

$$\frac{d^2}{dz^2} V_n + \frac{2}{z} \frac{d}{dz} V_n - \frac{n}{z^2} V_n = 0, \quad n = 0, 1. \quad (6.6)$$

The general solution to the latter equation is

$$V_n(|\mathbf{x} - \mathbf{y}|) = d_n |\mathbf{x} - \mathbf{y}|^{l_1^n} + c_n |\mathbf{x} - \mathbf{y}|^{l_2^n}, \quad n = 0, 1, \quad (6.7)$$

with  $d_n, c_n$  being constants, while  $l_1^n, l_2^n$  may be found as the roots of the equation

$$l^{n2} + l^n = n,$$

i.e.

$$l_1^n = -\frac{1 + \sqrt{1 + 4n}}{2}; \quad l_2^n = \frac{-1 + \sqrt{1 + 4n}}{2}. \quad (6.8)$$



It is easy to see that for  $n = 0$ , as  $d_0 = -1/4\pi$ , we get the *Coulomb-type potential*:

$$l_1^0 = -\frac{1 + \sqrt{1}}{2} = -1; \quad l_2^0 = \frac{-1 + \sqrt{1}}{2} = 0, \quad (6.9)$$

$$V_0(|\mathbf{x} - \mathbf{y}|) = -1/4\pi |\mathbf{x} - \mathbf{y}|^{-1} + c_0; \quad (6.10)$$

and for  $n = 1$ , the "golden section" potential for the *golden-section equation*

$$(l^1)^2 + l^1 = 1,$$

with

$$l_1^1 = -\frac{1 + \sqrt{5}}{2} \approx -1.618; \quad l_2^1 = \frac{-1 + \sqrt{5}}{2} \approx 0.618, \quad (6.11)$$

$$V_1(|\mathbf{x} - \mathbf{y}|) = -d_1|\mathbf{x} - \mathbf{y}|^{-1.618} + c_1|\mathbf{x} - \mathbf{y}|^{0.618}. \quad (6.12)$$

The latter, "golden section", potential (unlike the Coulomb-type one) implies the rearrangement of the naive perturbation series and the spontaneous breakdown of the chiral symmetry. In turn, this involves the constituent gluonic mass in the Feynman graphs: this mass changes the asymptotic freedom formula in the region of low transferred momenta; thus the coupling constant  $\alpha_{QCD}(q^2 \sim 0)$  becomes finite.

The "golden section" potential (6.12) may be also considered as an origin of "hadronization" of quarks and gluons in QCD [17, 23, 37, 38, 51].

Our assumption that the "golden section" potential (6.12) changes the infrared behaviour of Minkowskian QCD and gluodynamics has its roots in one very illustrative and herewith simpler example of increasing potentials.

Let us consider, following the papers [37, 38], the squared potential [17, 52]

$$V_R(r) = V_0 r^2; \quad V_0^{1/3} \sim 234 \text{ MeV}. \quad (6.13)$$

In the papers [37, 38] there was demonstrated, by application of definite numerical methods (there were the "shooting" [53] and the Runge-Kutta-Gill [54] methods) that the appearance of the squared increasing potential (6.13) [52] involves the following Modification of the gluonic propagator in the infrared momenta region [37, 38]:

$$D_{ij}^{mod}(k_0, k) = (\delta_{ij} - \hat{k}_i \hat{k}_j) \frac{1}{k_0^2 - \omega^2(\mathbf{k}) - i\epsilon}, \quad \hat{k}_i = \frac{1}{|\mathbf{k}|} k_i : \quad (6.14)$$

with

$$\begin{aligned} \underline{\omega}(k) &\longrightarrow \frac{2}{\underline{\mathbf{k}}^2}, \quad \mathbf{k} \rightarrow 0, \\ \underline{\omega}(k) &\longrightarrow \underline{\mathbf{k}} \quad \mathbf{k} \rightarrow \infty; \end{aligned} \quad (6.15)$$

where

$$\left( \frac{\underline{\omega}}{\underline{k}} \right) = (N_c V_0)^{-1/3} \left( \frac{\omega}{k} \right) \quad (6.16)$$

and  $N_c$  is the number of colours in the considered theory. This implies that gluons effectively acquire a structural mass depending on the momentum:

$$m_g(k^2) = \sqrt{\omega^2(k) - \mathbf{k}^2}, \quad (6.17)$$

such that  $m(0) \rightarrow \infty$ .

The modified gluonic propagator (6.14) vanishes in this limit.

On the other hand, in the ultra-relativistic region  $\mathbf{k} \rightarrow \infty$  we come to the standard gluonic propagator providing the asymptotic freedom of quarks.

It is also worth to cite here [37, 38] the "modified" formula for the running gluonic coupling constant  $\alpha_s^{mod}(q)$  (as a function of the transferred momentum  $q$ ) that follows from the formula (6.14) for the "modified" gluonic propagator in the presence of the square increasing potential  $V_R(r)$  [17, 52].

In the "world without quarks" i.e. when the number of flavours  $N_f = 0$ , the "modified" formula for the running gluonic coupling constant  $\alpha_s^{mod}(q)$  takes the look [38]

$$\alpha_s^{mod}(0) = \frac{1}{\beta \left[ 1 + \ln \left( \frac{4N_c V_0^{1/3}}{\Lambda} \right)^2 \right]} \approx 0.2; \quad \beta = \frac{11}{4\pi}; \quad (6.18)$$

at the zero value of the transferred momentum  $q$ . Herewith the ultraviolet cut-off parameter  $\Lambda$  is adopted [37] to be  $\Lambda \approx 110$  Mev.

There may be shown [37, 38] that  $\alpha_s^{mod}(q) \leq \alpha_s^{mod}(0) \lesssim 0.2$  in the whole permissible region of the transferred momentum  $q$  and, therefore, one may use the perturbation theory for all the transferred momenta  $q$ .

The precise computations about the gluonic propagator and gluonic coupling constant in the Minkowskian non-Abelian theory [8, 17, 23, 25] also demand the application of definite numerical methods, alike the methods [53, 54] that were applied [37, 38] in the case of the squared potential (6.13) [52]. We leave these computations out of our present discussion.

## 7 Feynman and FP path integrals.

In the "pure" Minkowskian YM theory [8, 17, 23, 25], without of the fermionic sector, the Feynman path integral over independent variables includes, in particular, the integration over the topological variable  $N(t)$ :

$$Z_F[J] = \int \prod_t dN(t) \tilde{Z}[J^U], \quad (7.1)$$

with

$$\tilde{Z}[J^U] = \int \prod_{t,x} \left\{ \prod_{a=1}^3 \frac{[d^2 A_a^{(0)} d^2 E_a^{(0)}]}{2\pi} \right\} \exp i \{ \mathcal{W}_{YM}(\mathcal{Z}) + \tilde{W}_{YM}(A_a^{(0)}) + S[J^U] \}. \quad (7.2)$$

As we have seen above, the functionals  $\tilde{W}$  and  $S$  are given in terms of variables containing non-perturbation phase factors  $U = U_{\mathcal{Z}}$ , (5.7), of the Gribov topological degeneration of initial data. These factors disappear in the action functionals  $W_{YM}$  and  $\tilde{W}_{YM}$  (due to their absorption [13] in the gauge invariant YM tensor squared,  $(F_{\mu\nu}^a)^2$ , on which the action functionals  $W_{YM}$  and  $\tilde{W}_{YM}$  explicitly depend), *but not in the source term*:

$$S[J^U] = \int d^4x J_i^a \bar{A}_a^i, \quad \bar{A}_i = U(\hat{A}^{(0)})U^{-1}. \quad (7.3)$$

This reflects the fact of the topological degeneration of physical fields in the Minkowskian YM model [8, 17, 23, 25].

Generally speaking, the phase factors  $U_{\mathcal{Z}}$  in the Minkowskian non-Abelian theory [8, 17, 23, 25], as a "relic" of the fundamental quantization by Dirac [26], remember all the information about the chosen reference frame, vacuum BPS (Wu-Yang) monopoles, rising "golden section" potential (6.12) of the instantaneous interaction between (two) total currents (5.20) and other initial data, including their topological degeneration and infrared confinement (see farther) [8].

The constraint-shell formulation distinguishes the *bare* "gluon", as a *weak deviation of the given vacuum monopole with the topological index  $n = 0$* , and the *observable (physical) "gluon" averaged over the topological degeneration* (i.e. *over all the Gribov copies*) [17, 30]:

$$\bar{A}^{phys} = \lim_{L \rightarrow \infty} \frac{1}{2L} \sum_{n=-L}^{n=+L} \bar{A}^{(n)}(\mathbf{x}) \sim \delta_{r,0}; \quad (7.4)$$

whereas in QED any constraint-shell field is a *transverse photon*.

A more detailed analysis of the latter formula will be performed in the next section.

We may speak that topological Dirac variables (4.32), involving the topological degeneration of initial states in a Minkowskian non-Abelian theory are a physical origin of hadronization and confinement, treated as non-local monopole effects.

These topological Dirac variables distinguish the unique gauge (to within the appropriate cohomological class).

In QED *it is the Coulomb gauge*, while in the Minkowskian YM theory [8, 17, 23, 25] *it is the covariant generalization of the covariant Coulomb gauge in the presence of vacuum BPS monopoles*. In the latter case takes place the Gribov ambiguity in the choice of the transverse Coulomb gauge [4, 24, 44] in all the topological sectors of the Minkowskian YM theory [8, 17, 23, 25] due to the nontrivial  $U(1) \rightarrow SU(2)$  embedding. In turns, this involves the infinite number of transverse YM fields in each topological sector of that theory.

When we go over to another (relativistic) gauge of physical sources, say  $F(A) = 0$ , at the level of the FP integral [20], all the monopole effects, including the topological degeneration of initial data and rising potential, may be lost (as the Coulomb potential is lost in QED in relativistic invariant gauges).

Recall that to prove the equivalence of the Feynman integral to the FP integral in an arbitrary gauge, we (see [17], §2.5) change variables:

$$A_k^*[A^F] = v[A^F](A_k^F + \partial_k)(v[A^F])^{-1}, \quad (7.5)$$

$$\psi^*[A^F] = v[A^F]\psi, \quad (7.6)$$

and concentrate all the monopole effects in phase factors before physical sources:

$$S^* = \int d^4x \left( (v[A^F])^{-1} \bar{s}^* \psi^F + \bar{\psi}^F (v[A^F])^{-1} s^* + J_i^* A_i^*[A^F] \right). \quad (7.7)$$

The change of sources removes all these effects [17].

Such change of sources was possible in the Abelian gauge theory only for scattering amplitudes [20] in neighbourhoods of poles of their Green functions, when all the particle-like excitations of fields are on their mass-shells (we recommend our reader to understand this fact with the "classical" example of electronic propagators [39]).

However, for the cases of non-local bound states in QED and QCD and other phenomena involving these fields *off their mass-shells*, the Faddeev theorem about the equivalence of different "gauges" (see, e.g., (7.23) in [21]) is not valid.

## 8 Free rotator: topological confinement.

The nontrivial topology may be an origin of the colour confinement Minkowskian non-Abelian gauge theories *via the complete destructive interference, in the infrared momenta region, of the phase factors of the topological degeneration of initial data*.

A good mechanical analogy of the topological degeneration of initial data is the free rotator, involving the appropriate action of a free particle alike to (4.62):

$$W(N_{out}, N_{in}|t_1) = \int_0^{t_1} dt \frac{\dot{N}^2}{2} I; \quad p = \dot{N}I, \quad H_0 = \frac{p^2}{2I}; \quad (8.1)$$

given on the ring, with the points  $N(t) + n$  ( $n \in \mathbf{Z}$ ) being physically equivalent (cf. (4.65)).

Instead of initial data  $N(t=0) = N_{in}$  in mechanics, in the space with a trivial topology, the observer of the "topological" rotator (4.62), (8.1) sees *a manifold of initial data*  $N^{(n)}(t=0) = N_{in} + n$ ;  $n \in \mathbf{Z}$  (we also may write down this manifold as  $N^{(n)}(\mathcal{H}(t_0))$  in the light of the said in Section 4.3 about the Gribov topological degeneration of non-Abelian initial data).

The observer does not know where the rotator is. It may be in the points  $N_{in}, N_{in} \pm 1, N_{in} \pm 2, \dots$ . Therefore he would *average* the wave function (cf. (7.4)):

$$\Psi(N) = e^{ipN} \quad (8.2)$$

over all the values of the topological degeneration, involving the  $\theta$ -angle measure  $\exp(in\theta)$ . As a result, he gets the "observable" wave function

$$\Psi(N)_{observable} = \lim_{L \rightarrow \infty} \frac{1}{2L} \sum_{n=-L}^{n=L} e^{in\theta} \Psi(N+n) = \exp\{i(2\pi k + \theta)N\}, \quad k \in \mathbf{Z}. \quad (8.3)$$

In the opposite case, when  $p \neq 2\pi k + \theta$ , the appropriate wave function (i.e. *the probability amplitude*) disappears:  $\Psi(N)_{observable} = 0$ , *due to the complete destructive interference*.

This corresponds to the usual description of a repeated phenomenon in mathematical statistics (we may treat zero as the mathematical expectation value of the set  $\mathbf{Z}$ ).

The consequence of this topological degeneration is that the part of values of momentum spectrum becomes *unobservable* in comparison with a trivial topology.

This fact may be treated as the *confinement* of those values that do not coincide with

$$p_k = 2\pi k + \theta, \quad 0 \leq \theta \leq \pi. \quad (8.4)$$

The observable spectrum also follows from the constraint of equivalence of the points  $N$  and  $N + 1$ :

$$\Psi(N) = e^{-i\theta} \Psi(N + 1), \quad \Psi(N) = e^{ipN}. \quad (8.5)$$

(the  $\theta$ -angle is, indeed, the eigenvector of the gauge transformation  $T_1|\theta\rangle = e^{i\theta}|\theta\rangle$  corresponding to the raise of the given topological number on unit:  $T_1|n\rangle = |n + 1\rangle$ ; this is valid in the Euclidean as well as in the Minkowski spaces).

As a result, we obtain the spectral decomposition of the Green function of the free rotator (8.1) (as the probability amplitude of the transition from the point  $N_{in}$  to the point  $N_{out}$ ) over the observable values of the spectrum (8.4):

$$G(N_{out}, N_{in}|t_1) \equiv \langle N_{out} | \exp(-i\hat{H}t_1) | N_{in} \rangle = \frac{1}{2\pi} \sum_{k=-\infty}^{k=+\infty} \exp \left[ -i \frac{p_k^2}{2I} t_1 + ip_k(N_{out} - N_{in}) \right]. \quad (8.6)$$

Using the connection with the Jacobian theta-functions [55]:

$$\Theta_3(Z|\tau) = \sum_{k=-\infty}^{k=\infty} \exp[i\pi k^2 \tau + 2ikZ] = (-i\tau)^{-1/2} \exp\left[\frac{Z^2}{i\pi\tau}\right] \Theta_3\left(\frac{Z}{\tau} \middle| -\frac{1}{\tau}\right), \quad (8.7)$$

we may represent the expression (8.6) as the sum over all the paths:

$$G(N_{out}, N_{in}|t_1) = \sqrt{\frac{I}{4\pi i t_1}} \sum_{n=-\infty}^{n=\infty} \exp[i\theta n] \exp[+iW(N_{out}, N_{in}|t_1)], \quad (8.8)$$

with

$$W(N_{out} + n, N_{in}|t_1) = \frac{(N_{out} + n - N_{in})^2 I}{2t_1} \quad (8.9)$$

being the rotator action (8.1).

## 9 Infrared quark confinement as a destructive interference of "large" Gribov topological multipliers.

The topological confinement similar to the complete destructive interference of the phase factors of the topological degeneration (*the latter one is a pure quantum effect*) may occur in classical non-Abelian field theories.

Recall that, at the time of the first paper by Dirac [26], the so-called "classical relativistic field theories" were found in the papers by Schrödinger, Fock, Klein, Weyl [56, 57] as patterns of *relativistic quantum mechanics*, i.e. as results of the primary quantization. The phases of the gauge transformations were introduced by Weyl [57] *as pure quantum magnitudes*.

The free rotator theory implies that the topological degeneration may be removed as all the Green functions are averaged over the values of the topological variable and all the possible angles of orientation of the monopole unit vector  $\mathbf{n}$  in the group space (instead of the instanton averaging over interpolations [3] between different vacua in the Euclidean space).

Averaging over all the parameters of degeneration may lead to the complete destructive interference of all the colour amplitudes [30, 31, 32, 45].

To verify the latter statement, we should recall that the Gribov matrices  $v^{(n)}(\mathbf{x})$  in (4.32), (4.34) have, indeed, the spatial asymptotic  $v^{(n)}(\mathbf{x}) \rightarrow 1$  as  $|\mathbf{x}| \rightarrow \infty$ , (4.50) [16, 45].

In effect, (4.50) found to be the normalization condition that would be imposed onto the Gribov matrices  $v^{(n)}(\mathbf{x})$  in the Minkowskian YM theory [8, 17, 23, 25] to ensure a good infrared behaviour of this theory.

In Section 4.3 we have ascertained the necessary look [45] of Gribov matrices  $v^{(n)}(\mathbf{x})$  for satisfying the normalization (4.50). As a result, we came to the spatial asymptotic (4.51)- (4.53) for Gribov matrices  $v^{(n)}(\mathbf{x})$  as  $|\mathbf{x}| \rightarrow \infty$ .

Now our task is to demonstrate that "large" Gribov matrices  $v^{(n)}(\mathbf{x})$ , possessing the spatial asymptotic (4.51)- (4.53), disappear, in the infrared limit  $|\mathbf{x}| \rightarrow \infty$ , in quark Green functions in all the orders of the perturbations theory.

The said quark Green functions may be obtained issuing from the FP path integrals formalism [45].

Let us fix a gauge, say

$$\det \hat{\Delta} = 0, \quad (9.1)$$

for the FP determinant  $\det \hat{\Delta}$  in (3.19).

It is highly obvious (see, e.g., the work [44]) that in the Minkowskian YM model [8, 17, 23, 25] the gauge (9.1) is mathematically equivalent to the Gribov ambiguity equation (4.39).

The standard FP path integral [20], as it is well known, has the look [17]

$$Z^*[s^*, \bar{s}^*, J^*] = \int \prod_{\mu} DA_{\mu}^F D\psi^F D\bar{\psi}^F \Delta_{FP}^F \delta(F(A^F)) e^{iW[A^F, \psi^F, \bar{\psi}^F] + S^*}, \quad (9.2)$$

involving the source term (7.7).

In particular, the gauge (9.1), specifying, in the Minkowskian YM theory [8, 17, 23, 25], the Gribov ambiguity in the choose of transverse YM fields: topological Dirac variables (4.32), implies the following look for the FP path integral (9.2) [21, 45]:

$$Z_{R,T}(s^*, \bar{s}^*, J^*) = \int DA_i^* D\psi^* d\bar{\psi}^* \det \hat{\Delta} \delta\left(\int_{t_0}^t d\bar{t} D_i(A) \partial_0 A^i\right)$$

$$\times \exp\{i \int_{-T/2}^{T/2} dt \int_{|\mathbf{x}| \leq R} d^3x [\mathcal{L}^I(A^*, \psi^*) + \bar{s}^* \psi^* + \bar{\psi}^* s^* + J_i^{*a} A_a^{i*}]\}. \quad (9.3)$$

In this formula the Lagrangian density  $\mathcal{L}^I$  corresponds to the constraint-shell action of the Minkowskian YM theory (or Minkowskian QCD) [8, 17, 23, 25] taking on the surface of the Gauss law constraint (3.5);  $R$  is an enough large real number, we may think that  $R \rightarrow \infty$ .

This involves the formal explicit dependence of the FP path integral (9.3) on Dirac variables  $A^*, \psi^*, \bar{\psi}^*$ .

The same cause implies the gauge fixation  $\delta(\int_{t_0}^t d\bar{t} D_i(A) \partial_0 A^i)$  in (9.3). This gauge is mathematically equivalent to the Coulomb transverse gauge (4.30), (4.38).

At the spatial infinity,  $|\mathbf{x}| \rightarrow \infty$  (i.e. in the infrared region of the momenta space), the general formulas (7.5), (7.6) [17] for (topological) Dirac variables inherent in the Minkowskian YM theory [8, 17, 23, 25] acquire the asymptotical look (4.51) [45].

Upon this substituting and averaging the FP path integral (9.3) over the Gribov topological degeneration of initial data, i.e. over the set  $\mathbf{Z}$  of integers, we get the generating functional that corresponds to the topological confinement in the Minkowskian YM theory [8, 17, 23, 25]:

$$Z_{conf}(s^*, \bar{s}^*, J^*) = \lim_{|\mathbf{x}| \rightarrow \infty} \lim_{T \rightarrow \infty} \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{n=-L/2}^{n=L/2} Z_{R,T}^I(s_{n,\phi_i}^*, \bar{s}_{n,\phi_i}^*, J_{n,\phi_i}^*), \quad (9.4)$$

with  $Z_{R,T}^I(s_{n,\phi_i}^*, \bar{s}_{n,\phi_i}^*, J_{n,\phi_i}^*)$  being the FP path integral (9.3) rewritten in terms of Gribov exponential multipliers (4.51) [45] at the spatial infinity.

It is worth to note a good consent between the generating functional (9.4) and Eq. (7.4) [17] for an observable (physical) gluon.

The variation of this generating functional by the sources:

$$\left(\prod_{\alpha=1}^3 \frac{\delta}{\delta s_{n,\phi_\alpha}^*}\right) \left(\prod_{\beta=1}^3 \frac{\delta}{\delta \bar{s}_{n,\phi_\beta}^*}\right) \left(\prod_{\gamma=1}^3 \frac{\delta}{\delta J_{n,\phi_\gamma}^*}\right),$$

is accompanied by averaging the appropriate Green functions over the Euler angles  $\phi_i$  ( $i = 1, 2, 3$ ) in (4.52). The latter ones describe the positions of colour degrees of freedom in the  $\{\phi_i\}$  reference frame. Herewith the Euler angles  $\phi_i$  would satisfy the normalization (4.54) to achieve the quested asymptotic (4.50) for Gribov exponential multipliers  $v^{(n)}(\mathbf{x})$ .

Let as compute the one-particle Green function for a quark in Minkowskian YM QCD [8, 17, 23, 25] involving fermionic degrees of freedom including as perturbation fields over the Minkowskian YM vacuum [25].

In the lowest order of the perturbation theory we then get [32, 45]

$$G(\mathbf{x}, \mathbf{y}) = \frac{\delta}{\delta s^*(x)} \frac{\delta}{\delta \bar{s}^*(y)} Z_{conf}(s^*, \bar{s}^*, J^*)|_{s^*=\bar{s}^*=0} = G_0(x-y) f(\mathbf{x}, \mathbf{y}), \quad (9.5)$$

with  $G_0(x - y)$  being the quark propagator in the perturbations theory and

$$f(\mathbf{x}, \mathbf{y}) = \lim_{|\mathbf{x}| \rightarrow \infty, |\mathbf{y}| \rightarrow \infty} \lim_{L \rightarrow \infty} (1/L) \sum_{n=-L/2}^{n=L/2} v^{(n)}(\mathbf{x}) v^{(n)}(-\mathbf{y}). \quad (9.6)$$

To estimate the function  $f(\mathbf{x}, \mathbf{y})$  [45], we should substitute the asymptotic expression (4.51) for Gribov exponential multipliers in the latter formula.

This results  $f(\mathbf{x}, \mathbf{y}) = 1$  due to the spatial asymptotic  $v^{(n)}(\mathbf{x}) \rightarrow 1$  as  $|\mathbf{x}| \rightarrow \infty$  for the Gribov topological multipliers (4.51), involving the normalization (4.54).

Thus we see that only "small" Gribov exponential multipliers  $v^{(0)}(\mathbf{x})$  contribute in  $f(\mathbf{x}, \mathbf{y})$  and, therefore, in the one-particle quark Green function (9.5).

The same result may be also demonstrated [45] in the momenta space. Upon substituting

$$t^0 = 0; \quad t^a = \frac{\tau^a}{r} \pi; \quad a = 1, 2, 3;$$

in (4.51) we get

$$G(p) = \lim_{L \rightarrow \infty} (1/L) \sum_{n=-L/2}^{n=L/2} \frac{1}{\hat{p} + \hat{t}n} = 0 \quad (\hat{p} = p_\mu \gamma^\mu) \quad (9.7)$$

for the one-particle quark Green function in the momenta space. It becomes  $O(1/p)$  in the limit  $n \rightarrow \infty$ . This confirms Eq. (9.6) valid in the coordinate space.

Consider now [45] a quark loop, treated as the vacuum expectation value of fermionic currents

$$\langle j^\Gamma(x), j^\Gamma(y) \rangle, \quad j^\Gamma = \bar{\psi} \Gamma \psi. \quad (9.8)$$

In the lowest order of the perturbation theory we then get

$$\begin{aligned} \langle j^\Gamma(x), j^\Gamma(y) \rangle_p = & \int d\Omega(\phi_x) d\Omega(\phi_y) \lim_{L \rightarrow \infty} (1/L) \sum_{n=-L/2}^{n=L/2} \text{tr} [v_{n, \phi_x}^{-1}(\mathbf{x}) \times \\ & \times \Gamma v_{n, \phi_x}(\mathbf{x}) G_0(x - y) v_{n, \phi_y}^{-1}(\mathbf{y}) \Gamma v_{n, \phi_y}(\mathbf{y}) G_0(y - x)], \end{aligned} \quad (9.9)$$

with  $d\Omega$  being the integral by Euler angles and  $\Gamma$  being a combination of Dirac matrices. We apply in the latter formula the denotation  $v_{n, \phi_x}(\mathbf{x})$  for Gribov exponential multipliers (4.51). This points out to their transparent connection with the Euler angles  $\phi_i$  ( $i = 1, 2, 3$ ).

When a matrix  $\Gamma$  is a colour scalar:  $v^{-1} \Gamma v = \Gamma$ , we deal with a quark loop with ordinary free propagators:

$$\langle j^\Gamma(x), j^\Gamma(y) \rangle_p = \text{tr} [\Gamma G_0(x - y) \Gamma G_0(y - x)], \quad (9.10)$$

whose imaginary part is different from zero.



But when a matrix  $\Gamma$  is "coloured", we come to expressions of the (9.5)- (9.7) type for quark loops.

For example, instead of the expression (9.7) for the one-particle quark Green function in the momentum representation, we then get for a quark loop the expression

$$\Pi(q) = \lim_{L \rightarrow \infty} (1/L) \sum_{n=-L/2}^{n=L/2} \int d^4x \text{tr}[\Gamma G_0(p+tn) \Gamma G_0(q-(p+tn))] = \int d^4x \text{tr}[\Gamma G_0(p) \Gamma G_0(q-p)], \quad (9.11)$$

in which takes place the complete cancellation of infinite-large momenta.

The alike topological confinement also takes place, in the Minkowskian QCD [8, 17, 23, 25] for YM (gluonic) fields.

As a result, only topological trivial gluons (7.4) [17] and topological trivial quarks (involving "small" Gribov multipliers  $v^{(0)}(\mathbf{x})$  in (7.6)) survive in the infrared momenta region. Just they form (as we shall demonstrate below) the observable (physical) hadronic bound states.

The similar mechanism of the infrared confinement was described for the two-dimensional model in the works [58, 59].

Upon averaging by infrared parameters of the Gribov topological degeneration, disappear all the quark Green functions which are not scalars under colour gauge transformations of the (4.32) type. On the other hand, colourless Green functions coincide with Green functions in ordinary perturbation QCD. They are similar to correlators between electromagnetic and weak currents. Thus we come to the quark confinement a la naive partonic model by Feynman [60].

In this case only *colourless* ("hadronic") states *form the complete set of physical states*. Using the example of a free rotator, (8.1), we see that the disappearance of the part of physical states due to the infrared topological confinement [45] does not violate the composition law for a Green function:

$$G_{ij}(t_1, t_3) = \sum_h G_{ih}(t_1, t_2) G_{hj}(t_2, t_3), \quad (9.12)$$

that is specified as the probability amplitude to find the system described by the Hamiltonian  $H$  in the state  $j$  in the time instant  $t_3$  when in the time instant  $t_1$  this system was in the state  $i$ , where  $(i; j)$  belong to the complete set of all the physical states  $\{h\}$ :

$$G_{ij}(t_1, t_3) = \langle i | \exp -i \int_{t_1}^{t_3} H | j \rangle. \quad (9.13)$$

A particular case of this composition law (9.12) is the unitarity of the QCD S-matrix:

$$SS^+ = I \implies \sum_h \langle i | S | h \rangle \langle h | S^+ | j \rangle = \langle i | j \rangle, \quad (9.14)$$

known as the law of the probability conservation for the S-matrix elements ( $S = I + iT$ ), where

$$\sum_h \langle i | T | h \rangle \langle h | T^* | j \rangle = 2\text{Im} \langle i | T | j \rangle \quad (9.15)$$

(cf. (64.2), (71.2) in [61]).

The left-hand side of this law is similar to the spectral series of the free rotator, (8.6). Herewith the infrared destructive interference [45] of Gribov "large" multipliers  $v^{(n)}(\mathbf{x})$  ( $n \neq 0$ ) keeps only the colourless "hadronic" states.

The right-hand side of the probability conservation law (9.15), far from resonances, may be represented by the perturbation series over the Feynman diagrams that follow from the Hamiltonian.

Due to the manifest gauge invariance of the fermionic Hamiltonian (Lagrangian) in (5.18) [25],

$$H[A^{(n)}, q^{(n)}] = H[A^{(0)}, q^{(0)}], \quad (9.16)$$

with  $q$  are being fermionic (quark) degrees of freedom. Herewith the record  $q^{(n)}$  for quark degrees of freedom implies that there also are Dirac variables having the general look (7.6).

The said implies that the Hamiltonian  $H[A^{(0)}, q^{(0)}]$  is invariant with respect to large gauge transformations (4.34) turning YM fields into physical topological Dirac variables.

Thus the Minkowskian QCD Hamiltonian does not depend on Gribov large multipliers  $v^{(n)}(\mathbf{x})$  ( $n \neq 0$ ) given in (4.47). We may treat this as *the complete destructive interference of Gribov factors of the topological degeneration in the QCD Hamiltonian*.

On the other hand, we come to the usual treatment of the colours confinement in QCD, when the colourless fermionic degrees of freedom  $q^{(0)}$  are considered as "hadronic" states, which we may identify with the Feynman partons [60].

The considered above holonomies theory (4.40)- (4.44) allow us to draw the conclusion that the colour confinement in the Minkowskian YM theory [8, 17, 23, 25], involving the Gribov equation (4.39) and its vacuum solution (4.46), is determined by the restricted holonomies group  $\Phi^0$  generated by the zero topological sector of this YM theory (more precisely, by the YM fields  $A^{(0)}$  of this sector satisfied the Coulomb gauge (4.30) and boundary conditions (4.50) at the spatial infinity) and representing the "small"  $U(1)$  gauge transformations). We may interpret this as the confinement criterion in Minkowskian YM QCD [8, 17, 23, 25] (that is also correct for the gluonic theory involving the  $SU(3)_{col} \rightarrow SU(2) \rightarrow U(1)$  spontaneous breakdown).

It turns out that (closely entangled each with other) the topological and quark confinements inherent in the Minkowskian model [8, 17, 23, 25] may be correct described in terms of the mixed task (see, e.g., §4.1 to Chapter 1 in [18]) to the Gribov ambiguity equation (4.39): the second-order differential equation in partial derivatives.

The said implies that we should supplement the initial Cauchy conditions (4.30), (4.25): in the zero topological sector, to the Gribov equation (4.39) (in the initial time instant  $t_0$ ) by the boundary condition (4.50) at the spatial infinity, involving the asymptotical look (4.51) [45] for Gribov topological multipliers  $v^{(n)}(\mathbf{x})$ .

It is highly appropriately to note here that the said mixed task for the Gribov equation (4.39) may found to be somewhat unexpected for our readers who devotes themselves to differential equation in partial derivatives.

It is well known that elliptical differential equation in partial derivatives, to which belongs the Gribov equation (4.39), need in none initial conditions: there are purely

stationary differential equation that do not depend on the time  $t$  (see, e.g., §4.1 to Chapter 1 in [18]).

But in the Minkowskian non-Abelian theory [8, 17, 23, 25] there is, for all that, the indirect dependence of the Gribov equation (4.39) on the time  $t$ .

This is associated with considering the Minkowskian non-Abelian theory [8, 17, 23, 25] on the surface of the YM Gauss law constraint (3.5). In this case the Dirac removal [26] of temporal YM components involves the Gauss law constraint (4.28), which we then resolve in the Coulomb transverse gauge (4.29), (4.30).

On the other hand, just the ambiguity in the choose of the Coulomb transverse gauge (4.30) is described by the Gribov equation (4.39) [4, 44]. The origin of this ambiguity in the definition of Dirac (topological) variables (4.34) apart from Gribov stationary multipliers  $v^{(n)}(\mathbf{x})$  [8].

Thus the both things: the YM Gauss law constraint (3.5), (4.29) and the Gribov ambiguity equation (4.39), are closely related each with other.

In effect, the appearance of the initial Cauchy conditions (4.30), (4.25) to the elliptical Gribov equation (4.39) is an original trace of the Minkowski space-time in the non-Abelian model [8, 17, 23, 25]. This involves the consideration of that model on the space-like surface  $\mathcal{H}(t_0)$  in the Minkowski space-time (in the initial time instant  $t_0$ ) where the Gribov topological degeneration of initial YM data occurs.

The QCD Hamiltonian  $H$  contains the perturbation series in terms only of zero degree of the map fields (i.e. in terms of *constituent colour particles*) that may be identified *with Feynman partons* [60].

The Feynman path integral of the (7.1) type, as the generating functional of these perturbations series, is an analogue of the sum over all the paths of the free rotator (8.8).

Therefore confinement, in the spirit of the complete destructive interference of colour amplitudes [17, 30, 32, 45], and the law of the probability conservation for the S-matrix, (9.15), imply the *Feynman quark-hadronic duality* that is the base of all the partonic models [60] and QCD applications [62].

The quark-hadronic duality gives the method of a direct experimental measurement of quark and gluonic quantum numbers issuing from the given deep-inelastic scattering cross-section [60].

For example, according to Particle Data Group, the ratio of the sum of the probabilities of the  $\tau$ -decay hadronic modes to the probability of the  $\tau$ -decay muonic mode is

$$\frac{\sum_h w_{\tau \rightarrow h}}{w_{\tau \rightarrow \mu}} = 3.3 \pm 0.3. \quad (9.17)$$

This is the left-hand side of Eq. (9.15) normalized to the value of the leptonic mode probability of the  $\tau$ -decay. On the right-hand side of Eq. (9.15) we have the ratio of the imaginary part of the sum over the quark-gluonic diagrams (in terms of constituent fields free from Gribov phase factors  $v^{(n)}(\mathbf{x})$ ) to the one of the leptonic diagrams. In the lowest order of perturbative QCD, on the right-hand side we get the number of colours  $N_c$ , therefore

$$3.3 \pm 0.3 = N_c. \quad (9.18)$$

Thus the degeneration of initial data in Minkowskian YM QCD [8, 17, 23, 25] may explain not only "*why we do not see quarks*", but also "*why we may measure their quantum numbers*".

The considered mechanism of confinement in Minkowskian YM QCD [8, 17, 23, 25], due to a quantum interference of the phase factors of the topological degeneration, disappears upon a change of the "physical" sources:  $A^* J^* \implies AJ$ , called the transition to another gauge in the gauge-fixing method [20].

This mechanism is a highly delicate thing, based (mathematically) on the just described mixed task (4.39) involving the initial conditions (4.30), (4.25) and asymptotical boundary condition (4.50) at the spatial infinity.

Thus it becomes obvious that the transition to another gauge from the transverse Coulomb one, (4.30), destroys the said mechanism of confinement in Minkowskian YM QCD [8, 17, 23, 25].

Instead of the hadronization and confinement, we obtain then, in the "relativistic" FP integral (9.2), only scattering amplitudes of free partons. But these amplitudes do not exist as physical observables in the Dirac quantization scheme depending on initial data.

Removing (topological) Dirac variables (4.32) (that are transverse and gauge invariant) via going over to another gauge, we herewith also remove the dependence of the FP integral (9.2) on a reference frame and initial data [17].

To understand the latter statement we should recall that Dirac variables are manifest relativistic covariant.

A good analysis of this feature of Dirac variables was carried out in the Polubarinov review [63] (see also Section 2.3 in [17]).

Thus going over to an arbitrary gauge (that is not transverse one) in the gauge-fixing method [20], we also remove the dependence of the gauge theory obtained in such a wise on a reference frame and initial data.

The said confirms the warning made by Schwinger [19] that gauges that are independent of a reference frame may be physically inadequate to the fundamental operator quantization; i.e. they may distort the spectrum of the original system <sup>12</sup>.

## 10 U(1)-problem.

The value of the vacuum chromo-magnetic field  $\langle B^2 \rangle$  may be estimated by the description of a process involving ABJ anomalies.

The simplest process of such type is the interaction of a pseudo-scalar bound state with an ABJ anomaly.

The potential source of ABJ anomalies in any QCD model are axial currents that follows, due to the variation principle, from the QCD Lagrangian, that always may be recast in such a wise to contain pseudoscalar combinations of fermionic fields involving the  $\gamma_5$  Dirac matrices.

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<sup>12</sup>"We reject all Lorentz gauge formulations as unsuited to the role of providing the fundamental operator quantization" [19].

We may recommend our readers several examples how to transform the QCD Lagrangian to result pseudoscalar combinations of fermionic fields. Such patterns of the appropriate transformations were demonstrated in the monograph [13], in §10.2, and in the papers [64, 65, 66].

In particular, axial currents may enter, as an item, the total current (5.20) [25], and this implies their contribution in the instantaneous current-current interaction (6.2).

Neglecting all the mesonic channels excepting  $\eta_0$  one, with [67]

$$\eta_0 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}, \quad (10.1)$$

one may incorporate the anomalous item [23, 25]

$$\tilde{W}_{\text{anomaly}}^{\eta_0}[\eta, \bar{\eta}] = C_\eta \int dt \bar{\eta}(t, 0) I_c \gamma_5 \eta(t, 0) \frac{g^2}{16\pi^2} \int d^3x F_{\mu\nu}^a * F_{\mu\nu}^a, \quad (10.2)$$

in the general (Minkowskian) QCD action.

Here  $\eta, \bar{\eta}$  are the fermionic sources,  $C_\eta$  is a constant we shall specify below.

At the level of Feynman diagrams this anomalous action corresponds [67] to the interaction between a pseudoscalar glueball field

$$\mathcal{Q} = \gamma_5 F_{\mu\nu}^a * F_{\mu\nu}^a \quad (10.3)$$

and two  $\eta_0$ -meson scalar states (10.1) attached to this glueball.

Due to (4.60),

$$\frac{g^2}{16\pi^2} \int d^3x F_{\mu\nu}^a * F_{\mu\nu}^a = \dot{N}(t). \quad (10.4)$$

Besides that, following the work [34], we utilize the normalization

$$\frac{g^2}{8\pi^2} \int d^3 D_i^{ab}(\Phi) \Phi_0^b B_i^a(\Phi) = 1 \quad (10.5)$$

for the vacuum magnetic field  $\mathbf{B}$ , specified by the Bogomol'nyi equation (2.12).

The physical anomalous term  $\tilde{W}_{\text{anomaly}}$ , (10.2), in the  $\eta_0$ -meson channel takes the look [68]

$$\tilde{W}_{\text{anomaly}}^{\eta_0}[\eta, \bar{\eta}] = C_\eta \int dt \eta_0 \dot{N}, \quad \eta_0(t) = \bar{\eta}(t) I_c \gamma_5 \eta(t). \quad (10.6)$$

Following Veneziano [68], we may identify  $\eta_0(t)$  with the field of the  $\eta_0$ -meson (10.1) at rest (multiplied onto  $\gamma_5$ ). The effective Minkowskian QCD action including the anomaly term (10.2) and the free rotator term, (4.62), is then [25]

$$W_{\text{eff}} = \int dt \left[ \frac{\dot{N}^2 I}{2} + \eta_0 C_\eta \dot{N} + \frac{1}{2} \dot{\eta}_0^2 V - \frac{\eta_0^2 m_0^2 V}{2} \right], \quad (10.7)$$

with  $m_0$  being the standard current quark mass contribution to the  $\eta_0$ -meson mass.

The effective action (10.7) is a particular case of the universal effective action inherent in gauge theories for descriptions of anomalous interactions [8, 17, 23]:

$$W_{eff} = \int dt \left\{ \frac{1}{2} (\dot{\eta}_M^2 - M_P^2 \eta_M^2) V + C_M \eta_P \dot{X}[A^{(N)}] \right\}, \quad (10.8)$$

with  $\eta_M$  being a bound state with the mass  $M_P$  in its rest reference frame and  $X[A^{(N)}]$  being the topological "winding number" functional.

In three-dimensional QED<sub>(3+1)</sub> this action, involving the constant [17]

$$C_M = C_{\text{positronium}} = \frac{\sqrt{2}}{m_e} 8\pi^2 \left( \frac{\psi_{Sch}(0)}{m_e^{3/2}} \right), \quad (10.9)$$

describes the decay of a positronium  $\eta_M = \eta_P$  into two photons associated with the "winding number" functional

$$\dot{X}_{QED}[A] = \frac{e^2}{16\pi^2} \int d^3x F_{\mu\nu}^* F^{\mu\nu} \equiv \frac{e^2}{8\pi^2} \int d^3x \varepsilon_{ijk} \dot{A}^i (\partial^j A^k - \partial^k A^j). \quad (10.10)$$

In one-dimensional QED<sub>(1+1)</sub> the action (10.8), involving the constant  $C_M = 2\sqrt{\pi}$  and the "winding number" functional

$$\dot{X}_{QED}(A^{(N)}) = \frac{e}{4\pi} \int_{-V/2}^{V/2} dx F_{\mu\nu} \epsilon^{\mu\nu} = \dot{N}(t) \Rightarrow F_{01} = \frac{2\pi \dot{N}}{eV}, \quad (10.11)$$

describes the mass of the Schwinger bound state  $\eta_P = \eta_{Sch}$  when the action (10.8) is added by the action of the Coleman electric field [31, 48]:

$$W_{QED} = \frac{1}{2} \int dt \int_{-V/2}^{V/2} dx F_{01}^2 = \int dt \frac{\dot{N}^2 I_{QED}}{2}, \quad (10.12)$$

where

$$I_{QED} = \left( \frac{2\pi}{e} \right)^2 \frac{1}{V}. \quad (10.13)$$

It is easy to see that the diagonalization of the total Lagrangian of the

$$L = \left[ \frac{\dot{N}^2 I}{2} + C_M \eta_M \dot{N} \right] = \left[ \frac{(\dot{N} + C_M \eta_M / I)^2 I}{2} - \frac{C_M^2}{2IV} \eta_M^2 V \right] \quad (10.14)$$

type implies the mass of a pseudo-scalar meson in QED<sub>(1+1)</sub>:

$$\Delta M^2 = \frac{C_M^2}{IV} = \frac{e^2}{\pi}. \quad (10.15)$$

In Minkowskian QCD<sub>(3+1)</sub> [8, 17, 23, 25] the diagonalization procedure similar to (10.14) implies an additional mass of the  $\eta_0$  meson:

$$L_{eff} = \frac{1}{2}[\dot{\eta}_0^2 - \eta_0^2(t)(m_0^2 + \Delta m_\eta^2)]V ,$$

$$\Delta m_\eta^2 = \frac{C_\eta^2}{I_{QCD}V} = \frac{N_f^2 \alpha_s^2 \langle B^2 \rangle}{F_\pi^2 2\pi^3} ,$$

with

$$I_{QCD} \equiv I = \left( \frac{2\pi}{\alpha_s} \right)^2 \frac{1}{V \langle B^2 \rangle} \quad (10.16)$$

founded from (4.63) [23].

This result allows us to estimate the value of the vacuum chromomagnetic field in Minkowskian QCD<sub>(3+1)</sub> [8]:

$$\langle B^2 \rangle = \frac{2\pi^3 F_\pi^2 \Delta m_\eta^2}{N_f^2 \alpha_s^2} = \frac{0.06 GeV^4}{\alpha_s^2} \quad (10.17)$$

by using estimating  $\alpha_s(q^2 \sim 0) \sim 0.24$  [25, 37].

Upon the computations we may remove the infrared regularization  $V \rightarrow \infty$ .

## Conclusion.

The principal problems of the discussion about stable vacuum states in any non-Abelian theory are the classes of functions and singularities.

These problems exist in all models of the QCD (YM) vacuum, including instantons [3, 4, 33, 13, 24] described by the delta-function-like singularities in the Euclidean space  $E_4$ .

Mysteries of nature are not only the actions and symmetries, but also the class of functions involving finite energies densities, applied in QFT (including QED) for description of physical processes.

If we explain any effect by these singularities, choosing a model of the nontrivial QCD (YM) vacuum, we should answer the questions: "Where are singularities of this vacuum from?" and "What is a physical origin of these singularities?"

We have presented here the model [8, 17, 23, 25] of the physical vacuum in the YM (gluonic) theory in the monopole class of functions involving the finite energies densities and without any singularity in a finite volume <sup>13</sup>, as a consequence ("smile") of the scalar Higgs field that disappears (like the Cheshire cat) from the spectrum of physical excitations of the considered theory in the limit of the infinite spatial volume.

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<sup>13</sup>Excluding, in effect, the infinite narrow cylinder with the diameter  $O(\epsilon)$  around the axis  $z$  in the chosen rest reference frame where Higgs vacuum BPS monopoles disappear [17, 25], while the vacuum "magnetic" field  $\mathbf{B}$  becomes infinite [6].

In other words, we have demonstrated that there exists a mathematically correct model of the YM (gluonic) vacuum, involving the finite physical energy-momentum spectrum in the Minkowski space, underlain by the Bose condensate of the Higgs scalar field in the limit of its infinite mass  $m/\sqrt{\lambda}$  (as the spatial volume  $V \rightarrow \infty$ ).

The initial  $SU(2)$  gauge symmetry in the Minkowskian YM theory [8, 17, 23, 25] is spontaneously broken down. This  $SU(2) \rightarrow U(1)$  breakdown always occurs in the presence of the Higgs  $SU(2)$  isovector.

As the Higgs field goes to the statistical (vacuum) expectation value at the spatial infinity, this implies the nontrivial topological structure of the residual symmetry group,  $U(1)$ , induced by the Higgs vacuum expectation value.

This nontrivial topological structure implies the presence of topological (magnetic) charges in this theory, i.e. *the inevitability of the monopole configurations of the Minkowskian YM vacuum with finite energies densities*.

We have considered the here presented theory in the BPS limit (2.11) [4, 6] when the self-interaction between the Higgs particles goes to zero. This allows us to consider Higgs particles (in the limit of their infinite number, i.e. at the level of statistical physics) as *an ideal gas*. We impose an additional condition to be stationary on this ideal gas (Bose condensate). This choice influences, in the end, the stationary nature of the monopole configurations of the YM vacuum.

Although the self-interaction between the Higgs particles goes to zero in the BPS limit (2.11), the system of YM and Higgs fields, actually present in the Minkowskian YM model [8, 17, 23, 25], is, indeed, a system of fields with the strong coupling determining by the YM (gluonic) coupling constant  $g$ .

This stipulates, in effect, all the features inherent in the Minkowskian YM model [8, 17, 23, 25], via the Bogomol'nyi equation (2.12), (2.14) depending on the coupling constant  $g$ .

As we have noted at the beginning of our discussion, the vacuum of such model, involving the strong coupling of gauge fields, is similar to the superfluid component in a helium II [5]. As in that case, long-range correlations of local excitations and cooperative degrees of freedom [16] also appear in the Minkowskian YM theory [8, 17, 23, 25].

The Bogomol'nyi equation (2.12), obtained at the evaluation of the lowest (*Bogomol'nyi*) bound  $E_{min}$ , (2.13), of the "(YM- Higgs)" energy, associated with vacuum monopole solutions (this bound depends on the Higgs mass  $m/\sqrt{\lambda}$ ), allowed us to specify monopole configurations of the Minkowskian YM vacuum as BPS or Wu-Yang monopoles (obtained as infinite spatial volume limits of BPS monopoles).

BPS monopoles (2.9), (2.10) (in the zero topological sector of the Minkowskian YM model [8, 17, 23, 25]) are regular in a finite spatial volume (although we cannot treat Higgs BPS monopole solutions (2.9) as completely regular ones: they diverge at the spatial infinity; on the other hand it is a large advantage, as far as we may treat Higgs vacuum solutions (2.9) as *a singular Bose condensate* [16], involving the appearance of vacuum "electric" monopoles (4.59)).

We have described the topological degeneration of initial data for monopole solutions belonging to the zero topological sector of the Minkowskian YM theory [8, 17, 23, 25].



This topological degeneration comes to Gribov copies of the covariant Coulomb gauge (4.30), treated as zero initial data for the YM Gauss law constraint (3.5) (this implies the absence of longitudinal YM fields in the initial time instant  $t_0$ ).

As far as YM fields are massless, the imposition of the transverse Coulomb gauge implies (see, e.g., [69]) the removal of temporal components of YM fields. We should necessarily remove these temporal YM components as long as they are non-dynamical degrees of freedom the existence of which contradicts the Heisenberg uncertainty principle [17, 26].

The removal a la Dirac [26] of temporal components of YM fields turns latter ones into topological Dirac variables (4.32) [8, 23, 25], transverse and physical. They are stationary and depend only on "large" (with topological numbers  $n \neq 0$ ) matrices  $v^{(n)}(\mathbf{x})$  of the Gribov topological degeneration (belonging to the nontrivial  $U(1) \rightarrow SU(2)$  embedding).

The Coulomb gauge (4.30) is not defined in the unique way. It is a purely non-Abelian effect [4, 44] called the Gribov ambiguity. To find the Gribov ambiguity in the choice of the Coulomb gauge (4.30), we should solve the Gribov ambiguity equation (4.39) of the second order.

Considering the Gribov topological degeneration in the given initial time instant  $t_0$  ensures that Dirac dressing matrices  $U^D$  [8], (4.36), remain trivial in this time instant, in despite of the Gribov ambiguity. Thus the Gribov ambiguity does not affect the nature of the Gribov topological degeneration.

In turns, this allows us to solve the Gribov ambiguity problem as the Cauchy task (4.39) with the initial conditions (4.28) (that is mathematically equivalent to (4.25)) and (4.30) in the time instant  $t_0$ , i.e. on the fixed space-like surface  $\mathcal{H}(t_0)$  in the Minkowski space-time. This means that we find topologically degenerated YM fields satisfying the Coulomb gauge (4.30), (4.38) (transverse and physical topological Dirac variables (4.32)) in the class of vacuum BPS monopoles and perturbation excitations over this monopole vacuum.

The Gribov equation (4.39) describes the nontrivial cohomological structure of YM fields in the Minkowski space. It turns out that there exists a one-to-one correspondence between the set of cohomologies classes of YM fields and the one of Gribov copies of the Coulomb gauge (4.30). This cohomological structure corresponds to the elements of the holonomies group  $H$  constructed over transverse YM fields belonging to the nontrivial  $U(1) \rightarrow SU(2)$  embedding.

The unit element of the holonomies group  $H$  (the whole this group is isomorphic to the residual gauge symmetry group  $U(1)$ ) is degenerated with respect to the class of exact 1-forms (with zero topological charges) induced by the Coulomb gauge (4.30) and Bogomol'nyi equation (2.12).

As a result, we may pick out the restricted holonomies subgroup  $\Phi^0$  in the holonomies group  $H$  isomorphic to the subgroup of "small"  $U(1)$  gauge transformations in the residual  $U(1)$  gauge group. We may treat the above nontrivial cohomological structure of YM fields as the solution of the Cauchy task (4.39), (4.30), (4.25).

YM fields are considered [8] as sums of vacuum fields (BPS monopoles) and weak perturbation excitations over this vacuum (multipoles). We suppose that multipoles possess

the same topological numbers that appropriate BPS monopoles.

The important point of our investigations is that the square of the vacuum expectation value of the "magnetic" tension,  $\langle B^2 \rangle$ , is different from zero in the Minkowskian YM model [8, 17, 23, 25]. This non-zero "magnetic" tension is an important distinction of the Minkowskian YM theory [8, 17, 23, 25] from the Euclidian one [3], where the zero asymptotic of the "magnetic" tension at the infinity [13] ensures the existence of instanton YM solutions.

We have shown that there exists the continuous topological variable  $N(t)$  specifying the zero mode of the Gauss law constraint (3.5) and depending on the time  $t$ ; it plays the role of the non-integer degree of the map.

The introduction of the topological variable  $N(t)$  results the appearance of vacuum "electric" monopoles (4.59) in the Minkowskian YM theory [8, 17, 23, 25], induced by temporal components of YM fields, (4.56), complementary to equal to zero (due the above resolving of the Gauss law constraint in terms of the Coulomb gauge (4.30)) ones in the "pure" YM theory. It is also the purely Higgs vacuum effect.

The calculations associated with the topological variable  $N(t)$  and vacuum "electric" monopoles (4.59) involve the action for the free rotator (4.62): with the rotary momentum  $I$  depending on  $\langle B^2 \rangle$  and real spectrum of momentum. This spectrum describes the collective solid potential rotation of the (YM-Higgs) vacuum.

We have constructed the complete Hamiltonian (4.66) [8] of the Minkowskian YM vacuum [8, 17, 23, 25]. It consists of two parts: "electric" and "magnetic" ones. This Hamiltonian is explicitly Poincare (in particular, CP) invariant, unlike the well-known  $\theta$ -term [13] in the instanton YM Euclidian theory [3]. Thus the CP problem may be solved in the Minkowskian YM theory [8, 17, 23, 25] in which vacuum YM and "electric" monopole modes appear.

The Coulomb gauge (4.30), (4.38) allows us to solve the Gauss law constraint (3.5) in terms of its zero mode (5.4), i.e. vacuum "electric" monopoles. As a result, the transverse "electric" tension (5.16) appears. It is again the purely Higgs effect.

On the other hand, the analysis of the constraint-shell action (5.11) allowed us to draw the following very important conclusion: that vacuum scalar BPS monopoles (and "electric" ones together with) disappear from the excitations spectrum in the infinite spatial volume limit  $V \rightarrow \infty$ . Nevertheless, Higgs BPS monopoles leave their trace in the unconstrained system (5.12) of local excitations as a weak "electric" tension  $\tilde{E}$ : a quantum fluctuation over the "electric" monopole.

This field plays a crucial role in the definition (5.20) [25] of the total current: the sum of the non-Abelian and fermionic components. We treat this current as an excitation over the physical (Minkowskian) YM vacuum.

The total current (5.20) satisfies Eq. (5.21) of the second order, depending on Wu-Yang (BPS) monopoles.

We have found the Green function of this equation as a composition of two potentials: the Coulomb type potential (6.10) and non-linear rising one ("the golden section potential") (6.12).

The latter potential implies the modified (in the infrared momenta region) gluonic

propagator, alike (6.14) [37, 38], and causes the hadronization of quarks.

The considered nontrivial topological structure of the vacuum in the Minkowskian YM theory [8, 17, 23, 25] may be in other non-Abelian Minkowskian theories. For instance, there is the spontaneous  $SU(3)_{col} \rightarrow SU(2)$  breakdown with the antisymmetric choice of the Gell-Mann matrices:  $\lambda_2, \lambda_5, \lambda_7$ , involving vacuum Wu-Yang monopoles (see Eqs. (3.24) -(3.25) in [23]). The essential point of the theory [23] was *the mix of world and group indices* at constructing vacuum Wu-Yang monopoles.

One may consider the behaviour of quarks in the Wu-Yang monopole background and compute the Green function of a quark (see (4.9)- (4.13) in [23]).

Arguments in favour of the non-Abelian Minkowskian theory [8, 17, 23, 25] are also the additional mass of the  $\eta_0$ -meson,  $\Delta m_\eta$ , and the infrared topological confinement [17, 30, 31, 32, 45].

So, speaking about the calculation about the additional mass of the  $\eta_0$ -meson in the non-Abelian Minkowskian model [8, 17, 23, 25] (this may be considered as the specific way to solve the  $U(1)$ -problem in the Minkowski space), it is worth to note the crucial role of (topologically trivial) Higgs BPS monopole modes (2.9) in forming the mesonic mass  $\Delta m_\eta$ .

Really, indeed

$$\Delta m_\eta \sim 1/\sqrt{I_{QCD}}.$$

On the other hand, the rotary momentum  $I_{QCD}$ , (4.63), of the non-Abelian Minkowskian vacuum is specifies by topologically trivial Higgs BPS monopole modes (2.9).

Moreover, this implies that the mesonic mass  $\Delta m_\eta$  is directly proportional to  $\langle B^2 \rangle \neq 0$ , the nonzero vacuum expectation value of the "magnetic" tension squared inherent in the non-Abelian Minkowskian model [8, 17, 23, 25] and induced by the Bogomol'nyi equation (2.12).

Discussing the infrared topological confinement [17, 30, 31, 32, 45] in the non-Abelian Minkowskian model [8, 17, 23, 25], we have demonstrated that "large" Gribov topological multipliers  $v^{(n)}(\mathbf{x})$  ( $n \neq 0$ ) disappear [45] in quark Green functions in all the orders of the perturbations theory in the limit of small transferred momenta.

Such effect is reached by virtue of the infrared spatial asymptotic (4.50) [45] of "large" Gribov topological multipliers  $v^{(n)}(\mathbf{x})$ . In turn, this asymptotic implies the normalization (4.54) for the Euler angles  $\phi_i$  ( $i = 1, 2, 3$ ).

We have demonstrated, recalling the arguments [45], that the infrared topological confinement implies the colour confinement in Minkowskian QCD [8, 17, 23, 25]. This means that only the colourless ("hadronic") states (Feynman partons) may be treated as physical states in that QCD. Minkowskian QCD [8, 17, 23, 25] is a unitary theory with respect to these states.

The criterion of the colours confinement in Minkowskian QCD [8, 17, 23, 25], involving the Coulomb gauge (4.30), is the existence of the nontrivial restricted holonomies group  $\Phi^0$  constructed on the transverse YM fields of the zero topological sector belonging to the  $U(1) \rightarrow SU(2)_{col}$  embedding. In addition these fields would satisfy the boundary condition (4.50) [45] to ensure the infrared topological (and colours) confinement in Minkowskian QCD [8, 17, 23, 25].

The said allowed us to describe the colour confinement in Minkowskian QCD [8, 17, 23, 25] in terms of the mixed task, in the space-like surface  $\mathcal{H}(t_0)$  in the Minkowski space-time, to the Gribov ambiguity equation (4.39).

This mixed task to the Gribov ambiguity equation (4.39) comes to the Cauchy task (4.39), (4.25), (4.30) (responsible for the topological degeneration of initial YM data in the Minkowskian theory [8, 17, 23, 25]) supplemented by the boundary condition (4.50) [45] to Gribov topological multipliers  $v^{(n)}(\mathbf{x})$  at the spatial infinity.

The Lorentz covariance of the considered theory may be carried out via Lorentz rotations of the time axis  $l_\mu^{(0)}$  along the complete momentum of each of the physical states, i.e. by transitions to reference frames where the initial data and spectra of these states are measured [23].

All these "smiles" of the Higgs scalar field disappear as we replace the fundamental Dirac variables [26, 70, 71] and change the gauge of their physical sources in order to obtain the conventional Faddeev-Popov integral [40] as a realization of the Feynman heuristic quantization [72]. We have made sure in this with the convincing example of the mixed task (4.39), (4.25), (4.30), (4.50) to the Gribov ambiguity equation (4.39) explained the colour confinement in Minkowskian non-Abelian theory [8, 17, 23, 25]. Going over from the Coulomb transverse gauge (4.30) to an arbitrary another one destroys this mixed task.

Such change removes, in general, all the time axes of the physical states, all the initial data, with their degeneration and destructive interference, and all monopole effects, including instantaneous interactions forming non-local bound states of the types of atoms in QED or hadrons in QCD.

In other words, the "smiles" of the Higgs field show us the limitedness of the FP heuristic path integral. The generalization of the Faddeev theorem of equivalence [20] (that is valid, indeed, only for local scattering processes) to the region of non-local processes removes both the initial data and the Laplace possibility of explaining (by these data) the non-local physical effects of the type of hadronization and confinement in this world.

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